<u>ARITHMATIC PROGRESSION(A.P)</u>

5.1:- <u>INTRODUCION</u>:-

In our daily life, we come across different situations where we need to work following a set pattern. For example, for a particular investment over the year we require an addition of fixed amount every year, so that we get a fixed amount for our use. An insect covers a particular distance every minute and distance keeps reducing every minute uniformly will the insect be able to reach destination? These are few daily life examples which need to be worked out and to work these, we will learn about the concept of sequences and in particular arithmetic sequence before that let's learn some terminology.

5.2:- <u>SEQUENCE</u>

A sequence is an arrangement of numbers in a definite order and according to some rule.

e.g. (a) 1,2,3,4,………. Is a sequence where each successive item is 1 greater than the preceding term.

(b) 1,4,9,16………..is a sequence where each term is the square of successive natural numbers and so on.

5.3:- <u>TERMS OF A SEQUENCE</u>:-

The different mumbers / number in a sequence are called terms.

e.g in the sequence 1,2,3,4……………., η,………………

1 is the Ist term and is denoted by a_1 or t_1 or T_1

2 is the 2nd term and is denoted by a_2 or t_2 or T_2

3 is the 3rd term and is denoted by a_3 or t_3 or T_3 and so on.

η is the ηth tern and is denoted by a_η or t_η or T_η.

General Term:- η^{th} tern of a sequence is called the General term of the sequence.

e.g in the sequence 1,4,9,16……….., η^2……..

i.e $1^2, 2^2, 3^2, 4^2$,………… η^2………

General term = n^{th} term = n^2 = a_n

If $a_1, a_2, a_3, \ldots\ldots\ldots a_n ,\ldots\ldots$ be a sequence, it can be resented by $<a_n>$ or $\{a_n\}$

5.4:- PROGRESSION:-   Sequence following definite patterns are called progression. E.g the following sequence are progression.

(i)   3,7,11,15,………   (ii)   ½, $^1/_5$, $^1/_8$, $^1/_{11}$,……….

(iii)   2,4,8,16……….

5.5:- <u>SERIES</u>:-   If the terms of the sequence are connected together by +ve or −ve sign, we get a series

For example

(i)   1+3+5+………….

(ii)   2+4+8+………….

(iii) $^1/_3$ + $^1/_6$ + $^1/_9$ +……… are all series of the Sequences

(i)   1,3,5,………….

(ii)   2,4,8,………….

(iii) $^1/_3$, $^1/_6$, $^1/_9$,……… respectively

5.6:- <u>FINITE SEQUENCE and INFINITE SEQUENCE</u>:-

If last term of a sequence is known, then it is a finite sequence. If last term is not known, then it is an infinite sequence.

e.g   1,2,3,5……………99 is a finite sequence.

2,4,6……… is an infinite sequence.

Note:- If we know any three consecutive terms of a sequence, we can find the rest of the terms of the sequence.

5.7:- TYPES OF PROGRESSION:-

There are four types of progression

(i)   Arithmatic progresson (AP)

(ii)   Geometric Progression(GP)

(iii)   Harmonic Progression(HP)

(iv)   Arithmatice Geometric prog (AGP).

(i) Arithmatic Progression (A.P). Sequence of no's such that the difference of any two successive terms of the sequence is a constant called common difference (C.D)

e.g   1,4,7,10,……….is an A.P with C.D= 3, and so on

(ii)   Geometric Progression (G.P):- Sequence of non-zero numbers such that the quotient (ratio) of any two successive terms of the sequence is a constant (called common Ratio (C.R)

E.g   2,4,8,16………… is a G.P with C.R = 2

(iii)   Harmonic Progression (H.P):-   Sequence of numbers such that their reciprocal form an Arithmatic progression (A.P)

e.g $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{9}$, ……..is a H.P, as the reciprocals of the terms of this progression i.e 3,6,9…………… from an A.P

(iv)   Arithmetic Geometric Progression (AGP):-   Sequence   of   nonzero numbers which is both an A.P as well as G.P . e.g 4,4,4,4………is an AGP as this sequence is both an AP with C.D zero and G.P as well with C.R equal to 1

5.8:-   ARITHMATIC PROGRESSION (A.P)

As already discussed a sequence ,finite or infinite is said to be an Arithmatic Progression (A.P), if the difference of a term and the previous

term is always same(constant)

Thus, a sequence $a_1, a_2, \ldots\ldots\ldots, a_n$ is said to form an A.P of

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \ldots\ldots\ldots\ldots = d$ (say)

i.e if $\boxed{a_n - a_{n-1} = }$ for n = 2,3,4 ……………

The constant 'd' (which is independent of n) is called the common difference (C.D)

of an A.P. the first term is usually denoted by 'a'.

e.g :- (i) 2,4,6,8 -------- is an A.P with a=2, d=2

(ii) $\frac{1}{6}, \frac{1}{3}, \frac{1}{2}$ --------- is an A.P with $a = \frac{1}{6}, d = \frac{1}{6}$

(iii) 0,- 3,-6,------is an A.P with a=0 . d= 3

5.9   General Term OR the nth term of An A.P :-

If 'a' and 'd' be the first term and C.D of an A.P, then the general form of an A.P

is   a, a+d, a+2d, a+3d,----------

i.e $a_1 = a = a+ (1 - 1)d$

$a_2 = a+d = a+(2 - 1)d$

$a_3 = a+2d = a+(3 - 1)d$

$a_4 = a+3d = a+(4 - 1)d$

—   —   —   —

—   —   —   —

$a_n = a+(n - 1)d$

Therefore, n^{th} term or the General term of A.P is $\boxed{a_n = a + (n - }$

Note: - If in the general term, we know any three quantities, then the fourth one can be easily determined by using above formula.

5.10:- n^{th} TERM FROM THE END OF AN A.P:-

To find the nth term from the end of the A.P

$a, a+d, a+2d, ------,(a+(n-3)d, a(n-2)d, a(n-1)d = l$

Where a= Ist term , d=C.D , l= last term , reverse the A.P, we get the reversed A.P as

l=a(n-1)d, a+(n-2)d, a+(n-3)d, ------,a+2d, a+d, a

in which ,now

First term (a)=l=a+(n-1)d, C.D (d) = -d and then we find the nth term from the beginning of the reversed A.P , we get

n^{th} term from end = l+(n-1)(-d)

=>
$$\boxed{n^{th} \text{ term from the end } = l-(n-1)d}$$

5.11   PROPERTIES OF AN A.P :-

 (i) If a constant is added/subtracted to each term of an A.P, the resulting sequence is also an A.P

(ii) if the term of an A.P are divided or multiplied by a non zero constant , then the resulting sequence is also an A.P

5.12   ARITHMETIC MEAN (A.M):- if three numbers a,A,b from an A.P , then 'A' is said to be an Arithmetic Mean (AM) of 'a' and 'b' and is given by

$$\boxed{A= \frac{a+b}{2}}$$

e.g. A.M of 4 and 16 is

$$A = \frac{4+16}{2} = \frac{20}{2} = 10$$

(i.e. 4 , 10,16 form A.P)

5.13 :- <u>SUM OF FIRST 'n' TERM OF AN A.P</u> :-

Let S_n = a+(a+d) + (a+2d) + - - - - - - - -+a+(n-1)d (= l)represents the sum of the first 'n' terms of an A.P with first term a, C.D=d and last term (l)= a+(n-1)d then

$S_n = \dfrac{n}{2}$ [2a+ (n-1) d] this form can be used when n, a and d are known

=> $S_n = \dfrac{n}{2}$ (a+a +(n-1)d)

=> $S_n = \dfrac{n}{2}$ (a+l)   we can use this form when we know the values of 'n' 'a' 'l'.

5.14 :- <u>TO FIND a_n when S_n of an A.P is given</u> :-

From given S_n, find S_{n-1} by substituting (n-1) instead of 'n' in S_n and then apply

$$\boxed{a_n = S_n - S_{n-1}}$$

If $S_n = n^2-1$, then

$S_{n-1} = (n-1)^2 - 1 = n^2 + \cancel{1} \cancel{2n} - \cancel{1}$

$= n^2 - 2n$

∴ $a_n = S_n - S_{n-1}$

=> $a_n = (n^2 - 1) - (n^2 - 2n) = (n^2 - 2n) = \cancel{n^2} - 1 - \cancel{n^2} + 2n$

=> $\boxed{a_n = 2n-1}$

5.15 :-   <u>TO SELECT THE TERMS OF AN A.P</u> :-

(i) When we have to select three terms in A.P , we take,

a— d, a, a+d

(ii) When we have to select four terms in A.P we take,

a-3d, a-d, a+d, a+3d

Note:- Gauss was the first mathematician who was associated with finding the sum of first 100 natural numbers.

(A) Multiple Choice questions

Choose the correct answer from the given four options:

1. In an A.P , if d=-4, n=7 , a_n= 4, then a is

    (A) 6 (B) 7 (C) 20 (D) 28

2. In an A.P , if a=3.5, d=0 , n= 101 then a_n will be

    (A) 0 (B) 3.5 (C) 103 (D) 104.5

3. The list of numbers -10, -6, -2, 2, - - - - -is

    (A) an A.P with d= -16

    (B) an A.P with d=4

    (C) an A.P with d=-4

    (D) not an AP

4. The 11th term of the AP:

    -5, -5/2, 0, 5/2, - - - - - - - is

    (A) -20 (B) 20 (C) -30 (D) 30

5. The first four terms of an AP, whose first term is -2 and common difference is -2 are

    (A) -2, 0, 2, 4

    (B) -2, 4, -8, 16

    (C) -2, -4, -6, -8

(D)-2, -4, -8, -16

6. The 21st term of the AP whose first two terms are -3 and 4 is

   (A) 17 (B) 137 (C) 143 (D) -143

7. If the 2nd term of an AP is 13 and the 5th term is 25, what is 7th term?

   (A) 30 (B) 33 (C) 37 (D) 38

8. Which term of the AP: 21, 42, 63, 84, - - - - ---- is 210?

   (A) 9th (B) 10th (C) 11th (D) 12th

9. If the common difference of an AP is 5, then what is $a_{18}-a_{13}$?

   (A) 5 (B) 20 (C) 25 (D) 30

10. What is common difference of an AP which is $a_{18}-a_{14}=32$?

    (A) 8 (B) -8 (C) -4 (D) 4

11. Two AP`s have the same common difference .the first term of one of these is -1 and that of other is -8. Then the difference between their 4th terms is

    (A) -1(B) -8 (C) 7 (D) -9

12. If 7 times the 7th term of an AP is equal to 11th times, its 11th term, then its 18th term will be

    (A) 7(B) 11 (C) 18(D) 0

13. The 4th term from the end of AP: -11, -8, -5, - - - - -- 49 is

    (A) 37(B) 40 (C) 43 (D) 58

14. The famous mathematician associated with finding the sum of first 100 natural numbers is

    (A) Pythagoras (B) Newton (C) Gauss (D) Euclid

15. If the first term of an AP is -5 and the common difference is 2, then the

sum of first 6 terms is

(A) 0 (B) 5 (C) 6 (D) 15

16. The sum of first 16 terms of the AP: 10, 6,2,-----is

(A) -320 (B) 320 (C)-352 (D) -400

17. In an AP if a=1, a_n =20 and S_n =399, then n is

(A) 19 (B) 21 (C) 38 (D) 42

18. The sum of first five multiples of 3 is

(A) 45 (B) 55 (C) 65 (D) 75

19. The 10th term of the AP: 5, 8, 11, 14,--------is

(A) 32 (B) 35 (C) 38 (D) 185

20. In an AP if a= -7.2, d= -3.6, a_n = 7.2 then n is

(A) 1(B) 3 (C) 4 (D) 5

Short Answer Questions

1. Find a, b and c such that the following numbers are in AP: a, 7, b, 23, c.

2. Determine the AP whose fifth term is 19 and difference of eighth term from the thirteenth term is 20,

3. The 26th, 11th and last term of an AP are 0, 3 and -1/5, respectively. Find the common difference and the number of terms

4. The sum of 5th and 7th terms of AP is 52 and 10th term is 46.Find the AP.

5 . Find the 20th term of AP whose 7th term is 24 less than the 11th term ,first term being 12 .

6. If the 9th term of an AP is zero, prove that its 29th term is twice its 19th term

7. Find whether 55 is a term of AP : 7, 10, 13,-------or not. if yes find which term

8. Determine k so that k^2+4k+8, $2k^2+3k+6$, $3k^2+4k+4$ are consecutive term of an AP.

9. Split 207 into three parts such that these are in AP and product of the two smaller parts is 4623. (Main concept staking 3 terms as a-d, a, a+d with their sum = 207)

10. The angles of a triangle are in AP .the greatest angle is twice the least .Find all the angles of the triangle

    [Hint sum of interiors <'s = 180. Taking d 3 <'s as a-d, a, a+d]

11. If the nth term of the two AP`s : 9,7 ,5, ----- and 24, 21, 18,---- are the same .find the value of n. Also find that term.

12. If the sum of 3rd and 8th terms of an AP is 7 and sum of 7th and 14th terms is -3,

    Find the 10th term

13. Find the 12th term from the end of the AP: -2, -4,-6,--0-------,-100

14. Which term of the AP : 53.48,43, ------ is the first negative term?

    [Hint let a_n be first −ve term ∴ $a_n < 0$]

15. How many numbers lie between 10 and 300, which when divided by 4 leave a remainder 3?

    [required AP is 11, 15, 19,……………299]

16. Find the sum of the two middle terms of the AP: -4/3, -1, -2/3, -------- $4\frac{1}{3}$

    [hint find no. of terms then median of n] n = 18 9th (+) 10th = 3

17. The first term of an AP is -5 and the last term is 45. If the sum of the terms of the AP is 120, then find the number of terms and the common difference.

18 Which term of the AP, -2,-7, - 12…..will be -77? Find the sum of this AP upto the term − 77

19. if a_n = 3-4n, show that

    a_1, a_2, a_3 ……form an AP. Also find S_{20}

20. In an AP, if S_n = n (4n + 1) find the AP   [Hint $a_n = S_n - S_{n-1}$] ,

21. In an AP, if Sn = $3n^2$ + 5n and ak = 164. Find the value of K.

22. If Sn denotes the sum of first n terms of an AP, prove that

    $S_{12} = 3(S_8 - S_4)$

23. Find the sum of first 17 terms of an AP whose 4th and 9th terms are − 15 and − 30. respectively

24. If sum of first 6 terms of an AP is 36 and that of the first 16 terms is 256. Find the sum of first 10 terms.

25. Find the sum of all 11 terms of an AP whose middle most term is 30.

26. Find the sum of last ten terms of the AP. 8, 10, 12,…..126.

27. Find the sum of first seven numbers which are multiples of 2 as well as of 9.

    [Hint: Take the LCM of 2 and 9]

28. How many terms of the AP:
    -15, -13, -11, ….. are needed to make the sum -55? Explain the reason for double answer.

29. If the numbers n − 2, 4n − 1 and 5n + 2 are in AP, Find the value of n.

30. Find the value of the middle most terms (s) of the AP: -11, --7, -3,…..49.

[C]   Long Answer Questions

1.   The sum of the first five terms of an AP and the sum of the first seven terms of the same AP is 167. If the sum of the first ten terms of this AP is 235, find the sum of its twenty terms.

2.   Find the

(i)   sum of those integers between 1 and 500 which are multiples of 2 as well as of 5.

(ii)   sum of those integers from 1 to 500 which are multiples of 2 as well as of 5

(iii)   sum of those integers from 1 to 500 which are multiples of 2 or 5

[Hint (iii) : These numbers will be: multiples of 2+ multiples of 5- multiples of 2 as well as of 5]

3.   The eighth term of an AP is half its second term and the eleventh term exceeds one third of its fourth term by 1. Find the 15th term.

4.   An AP consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the last three is 429. Find the AP.

5.   Find the sum of the integers between 100 and 200 that are

(i)   divisible by 9

(ii)   not divisible by 9

[Hint (ii): These numbers will be: Total numbers – Total numbers divisible by 9)

6.   The ratio of the 11th term to the 18th term of an AP is 2:3. Find the ratio of the 5th term to the 21st tem, and also the ratio of the sum of the first five terms to the sum of the first 21 term.

7.   Solve the equation

- 4 + (-1)+2+…..+ x = 437   [Hint take a_n = x, then s_n = 437]

8. Jaspal Singh repays his total loan of Rs 118000 by paying every month starting with the first installment of Rs 1000. If he increase the installment by Rs 100 every month. What amount will be paid by him in the 30th installment? What amount of loan does he still have to pay after the 30th installment?

9. The students of a school decided to beautify the school on the Annual Day by fixing colourful flags on the straight passage of the school. They have 27 flags to be fixed at interval of every 2 m. The flags are stored at the position of the middle most flag. Ruchi was given the responsibility of placing the flags. Ruchi kept her books where the flags were stored. She could carry only one flags at a time. How much distance did she cover in completing this job and returning back to collect her books? What is the maximum distance she travelled carrying a flag?

ANSWERS

Multiple Choice Question

Question	Ans	Question	Ans	Question	Ans	Question	Ans	Question	Ans

01	D	02	B	03	B	04	B	05	C
06	B	07	B	08	B	09	C	10	A
11	C	12	D	13	B	14	C	15	A
16	A	17	C	18	A	19	A	20	D

Short Question Answers

01. a = -1, b +15 c =31

02. 3, 7, 11, ----------

03. d = -1/5, n = 27

04. 1 , 6 , 11,16...........

05. 126

06. _____

07. Yes, 17th term

08. K = 0

09. 67,69,71

10. 40°, 60°, 80°,

11. n=16, term = - 21

12. a_{10}= -1

13. a_{12} = - 78

14. a_{12} = -2

15. n = 73

16. 3

17. (d = 10, n = 6)

18. [n = 16, S_{16} = -632]

19. [S_{20} = -780]

20. 5, 13, 21, 29.......

21. K = 27

22. _____

23. . S_{17} = -510

24. S_{10} = 100

25. S_{11} = 330

26. (S_{10} = 1170)

27. (S_7 = 504)

28. n = 11, 5

29. n=1

30. (n = 16, 8th , 9th)

Long Answer Question

01. 970

02. i)12250, ii) 12750, iii) 75250

03. 3

04. 3,7,11

05. i) 1683 , ii) 13167

06. $a_5:a_{21}$ = 1:3

$S_5: S_{21}$ = 5: 49

07. n = 50

08.   (Rs 44500)]

09.   (26 m)