SURFACE AREA AND VOLUMES

INTRODUCTION:

In our day-to-day life, we need a number of solids which are either a part of solid or a combination of these solids. We deal with cubes, cuboids, Cylinders, cones etc. In this chapter, we have to deal with surface area and volumes of various solids and also with the conversion of a solid into another.

Surface area refers to the area of the exposed surface of the three dimensional solid. There are two types of surface area.

1. Lateral Surface Area
2. Total Surface Area.

In general, lateral surface area does not include the base of the shape while the total surface is the area is the area of the entire object. Lateral surface area (lateral also means side), does not include the area of the top and bottom.

The volume of a solid refers to the three-dimensional space it occupies, often quantified numerically. One dimensional figure, namely lines and two-dimensional shapes, namely squares etc. are assigned zero volume in the three-dimensional space.

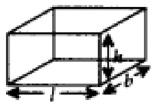
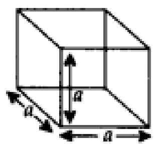
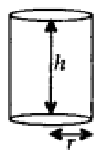
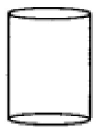
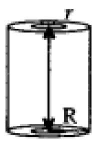
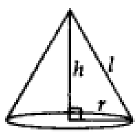
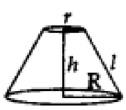
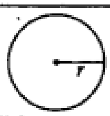
So, the surface area is the area that describes the material that will be used to cover a geometric solid. When we determine the surface areas of a geometric solid, we take the sum of the areas for each geometric form within the solid and the volume is a measure of how much a figure can hold. Surface area is measured in square units and volume is measured in cubic units.

Surface area is important to know situations where we want to wrap something, paint something and eventually while building things to get the best possible design. Finding the volume of an object can help us to determine the amount required to fill that object, like the amount of water needed to fill a bottle, an aquarium or tank.

For public works and industrial development activities, need of converting a solid into another solid of different shape or more than one solid of similar shape but with reduced size arises. When you convert a solid shape to another, its volume remains same, no matter how different the new shape is. In-fact, if one big cylindrical candle is melted to five small cylindrical candles, the sum of volume of the smallest candles is equal to volume of the bigger candle. Even if the conversion is to a different shape, the volume remains unchanged.

"The invention of surface area and volume is credited to Archimedes."

TABLE FOR SURFACE AREA AND VOLUME

Solid	Figures	Curved surface area (1)	Plane area (2)	Total area [1 + 2]	Volume	Remarks
Cuboid		Also known as lateral surface area $= 2(lh + bh)$	Area of: Top face $= lb$ Bottom face $= lb$ $\therefore lb + lb = 2lb$	$2(lb + bh + hl)$	$l.b.h$	l : length b : breadth h : height
Cube		Lateral surface area $= 4a^2$	Area of: Top face $= a^2$ Bottom face $= a^2$ $\therefore a^2 + a^2 = 2a^2$	$4a^2 + 2a^2 = 6a^2$	a^3	a : Side of cube
Right circular cylinder closed at top		Curved surface area $= 2\pi rh$	Area of: Top face $= \pi r^2$ Bottom face $= \pi r^2$ $\therefore \pi r^2 + \pi r^2 = 2\pi r^2$	$2\pi r^2 + 2\pi rh$ Or, $2\pi r(r + h)$	$\pi r^2 h$	r : radius h : height of cylinder
Right circular cylinder open at top		Curved surface area $= 2\pi rh$	Area of: Top face $= 0$ Bottom face $= \pi r^2$ $\therefore 0 + \pi r^2 = \pi r^2$	$2\pi rh + \pi r^2$ Or, $\pi r(2h + r)$	$\pi r^2 h$	r : radius h : height of cylinder
Hollow cylinder (Pipe)		$2\pi Rh$ • External surface area $= 2\pi Rh$ • Internal surface area $= 2\pi rh$	Area of: Top face $= \pi(R^2 - r^2)$ Bottom face $= \pi(R^2 - r^2)$	$2\pi Rh + 2\pi rh + 2\pi(R^2 - r^2)$	$\pi R^2 h - \pi r^2 h$ (External Vol. – Internal Vol.)	R : Radius of outer base r : radius of inner base h = height
Cone		πrl	Area of: Bottom Face $= \pi r^2$	$\pi r^2 + \pi rl$ Or, $\pi r(r + l)$	$\frac{1}{3}\pi r^2 h$	h = height of cone r = radius of cone l = slant height $= \sqrt{h^2 + r^2}$
Frustum		$\pi l(R + r)$	Area of: Top Face $= \pi r^2$ Bottom Face $= \pi R^2$	$\pi r^2 + \pi R^2 + \pi l(R + r)$	$\frac{1}{3}\pi h \cdot (R^2 + r^2 + Rr)$	h = height of frustum r = radius of top face R = Radius of base l = slant height
Sphere		$4\pi r^2$	None	$4\pi r^2$	$\frac{4}{3}\pi r^3$	r : radius of sphere
Hemisphere		$2\pi r^2$	πr^2	$3\pi r^2$	$\frac{2}{3}\pi r^3$	r : radius of hemisphere
Spherical shell		$4\pi R^2$ (Outer) $4\pi r^2$ (Inner)	None	$4\pi R^2 + 4\pi r^2$	$\frac{4}{3}\pi \cdot (R^3 - r^3)$	R:Radius of outer shell r:Radius of inner shell

1 Mark Question

Q.1 **If the radius and height of a cylinder are in the ratio 5:7 and its volume is 550 cm^3. Then its radius is equal to ($\pi = \frac{22}{7}$).**

a) 6 cm b) 7 cm c) 5 cm d) 10 cm

Q.2 **If the curved surface area of a solid right circular cylinder of height "h" and radius "r" is one-third of its total surface area, then**

a) $h = \frac{1}{3} r$ b) $h = \frac{1}{2} r$ c) h = r d) h = 2r

Q.3 **A hollow cylindrical pipe is 21 cm long. It its outer and inner diameters are 10 cm and 6 cm respectively, then the volume of the metal used in making the pipe is ($\pi = \frac{22}{7}$).**

a) 1048 cm^3 b) 1056 cm^3 c) 1060 cm^3 d) 1064 cm^3

Q.4 **A cone is within the cylinder and cylinder is within a cube touch, by all vertical faces with same base and height, then the ratio of their volumes will be**

a) 14:11:13 b) 44:33:11 c) 56:36:22 d) None of these

Q.5 **How many cubes each of edge 6cm can be cut from a cuboid of 42 cm x 36 cm x 24 cm:**

a) 124 b) 142 c) 168 d) 186

Q.6 **A medicine capsule in the shape of a cylinder of diameter 0.5 cm with two hemispheres stuck to each of its ends. The length of the entire capsule is 2 cm. The capacity of the capsule is:**

a) 0.36 cm^3 b) 0.35 cm^3 c) 0.34 cm^3 d) 0.33 cm^3

Q.7 **A solid piece of iron in the cuboid form of dimensions 49 cm x 33 cm x 24 cm, in moulded to form a solid sphere. The radius of the sphere is**

a) 21 cm b) 23 cm c) 25 cm d) 19 cm

Q.8 **The radius and the height of a right circular cone are in the ratio 5:12. If its volume is 314 cm^3, the slant height and radius are:**

a) 12, 5 cm b) 13,4 cm c) 1, 4 cm d) 13, 5 cm

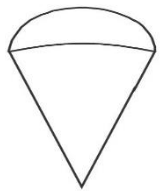
Q.9 A mason constructs a wall of dimension 270 cm x 300 cm x 350 cm with the bricks each of size 22.5 cm x 11.25 cm x 8.75 cm and it is assumed that 1/8 space is covered by the mortar. Then the number of bricks used to construct the wall is:

a) 11100 b) 11200 c) 11000 d) 11300

Q.10 A cone is cut though a plane parallel to its base and then the cone that is formed on one side of that plane is removed. The new part that is left over on the other side of the plane is called:

a) A frustum of a cone b) Cone c) Cylinder d) Sphere

Q.11 Total surface area of a lattu (top) as shown in figure is the sum of total surface area of hemisphere and the total surface of the cone (T/F)



Q.12 The total surface area of a hemisphere of radius is 2 r^2 (T/F)

Q.13 Two solid metallic right circular cones have same height, the radii of their base are r_1 and r_2. The two cones are melted together and recast into a right circular cylinder of height "h" then the radius of the base of the cylinder is $\sqrt{\frac{1}{2}(r_1^2 + r_2^2)}$ (T/F)

Q.14 The lateral surface area of a cuboid of length "ℓ", breadth "b" and height "h" is 2 (ℓ+ b) x h (T/F)

Q.15 If the height of a cylinder doubles, then the volume of a cylinder increases by a factor of 8 (T/F)

Q.16 A cube with side length 10 cm would fit inside a sphere with diameter 10 cm (T/F)

Q.17 If the height of the cone is double and the base remains the same. The volume of the cone doubles (T/F)

Q.18 The total surface area is ………….. for a cube of edge length "a"

Q.19 ………….. is the surface area of a solid cylinder and radius 2 cm and height 10 cm in (mm^2)

Q.20 A cylindrical pencil sharpened at one end is the combination of ………….. and …………..

Q.21 During conversion of solid from one sphere to another, the volume of the new shape will …………..

Q.22 The surface area of a sphere is equal to the curved surface area………………

ANSWER

Q.1	(a)	Q.2	(c)	Q.3	(b)	Q.4	(b)	Q.5	(c)	Q.6	(a)
Q.7	(a)	Q.8	(d)	Q.9	(b)	Q.10	(a)	Q.11	(T)	Q.12	(F)
Q.13	(F)	Q.14	(T)	Q.15	(F)	Q.16	(F)	Q.17	(T)	Q.18	($6a^2$)

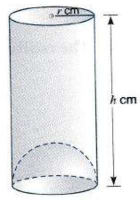
Q.19 (15072) Q.20 (Cone, cylinder) Q.21 Remain unaltered Q.22 Cylinder

2 MARKS QUESTIONS

Q.1 Three cubes each of side 5 cm are joined end to end. Find the surface area of the resulting cuboid.

Q.2 Define frustum of the cone.

Q.3 What is the capacity of a cylindrical vessel with a hemispherical portion raised upward to the bottom?



Q.4 A solid ball is exactly fitted inside the cubical box of side “a”. What is the volume of the ball?

Q.5 A rectangular solid metallic cuboid 9 cm x 8 cm x 2 cm is melted and recast into solid cubes each of side 2 cm. How many solid cubes can be made?

Q.6 The radii of the ends of a frustrum of a cone 40 cm high are 20 cm and 11 cm. Find its slant height.

Q.7 Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of hemisphere?

Q.8 A cone, a hemisphere and the cylinder stand on equal bases and have the same height. What is the ratio of their volumes?

Q.9 Match the Column

S.No	Type	Measurement
1	Volume of frustrum of cone	$\sqrt{h^2 (r_1 - r_2)^2}$
2	Slant height of frustrum of a cone	$\frac{1}{3}\pi h(r_1^2+r_2^2+r_1r_2)$
3	Curved surface area of frustrum of a cone	$\pi\ell(r_1+r_2)+\pi(r_1^2+r_2^2)$
4	Total surface area of a frustrum of a cone	$\pi\ell(r_1+r_2)$

Q.10 12 Spheres of same size are made from melting a solid cylinder of diameter 16 cm and 2 cm height. What is the diameter of each sphere?

Q.11 A cylinder has 5 cm height and 3 cm diameter. Find its total surface area.

Q.12 Find the curved surface area of a right circular cone whose slant height is 10 cm and the radius is 7 cm.

Q.13 Find the total surface area of a hemisphere of radius 21 cm.

Q.14 Give two point of difference between surface area and volume of a solid.

Q.15 Find the amount of water displaced by a solid spherical ball of diameter 28 cm.

ANSWER

Q.1	(350 cm^2)	Q.3	($\frac{\pi r^2}{3}$(3h-2r))	Q.4	$\frac{\pi a^3}{6}$	Q.5	(18)
Q.6	(41 cm)	Q.7	(9 units)	Q.8	(1:2:3)	Q.10	(2 cm)
Q.11	(61.23 cm^2)	Q.12	(220 cm^2)	Q.13	(4158 cm^2)	Q.15	(11498 $\frac{2}{3}$ cm^3)

3 Mark Questions

Q.1 The side of a solid metallic cube is 50 cm. The cube is melted and recast into 8000 equal solid cubical dice. Determine the side of the dice.

Q.2 A right circular cone of metal is 2.7 cm high and radius of base is 1.6 cm. It is melted and recast into a sphere. Find the radius of the sphere.

Q.3 The diameter of a metallic sphere is 6 cm. The sphere is melted and drawn into a wire of uniform circular cross section. If the length of the wire is 36 m. Find the radius of the cross section.

Q.4 How many balls each of radius 0.5 cm can be made from a solid sphere of metal of radius 10 cm by melting the sphere.

Q.5 A copper rod of diameter 1 cm and length 8 cm is drawn into wire of length 32 m of uniform thickness (diameter). Find the thickness of the wire.

Q.6 A shot putt is a metallic sphere of radius 4.9 cm. If the density of the metal is 7.8 g per cm^3. Find the mass of the shot putt.

Q.7 The volume of a right circular cone is 9856 cm^3. If the diameter of the base is 28 cm. Find

(a) height of the cone (b) Slant height of the cone

Q.8 A right triangle ABC with sides 5 cm, 12 cm and 13 cm is revolved about the side 12 cm. Find the volume of the solid so obtained.

Q.9 The diameter of the moon is approximately one fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

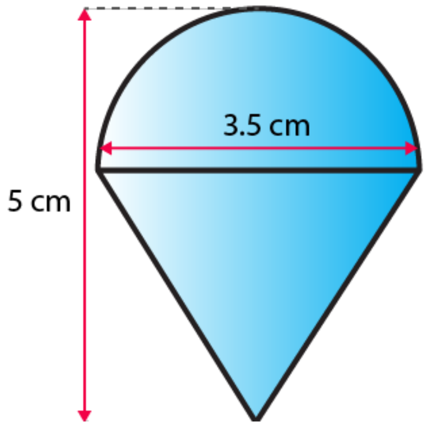
Q.10 The diameter of a metallic ball is 4.2 cm. What is the mass of the ball, if the density of metal is 8.9 g per cm^3?

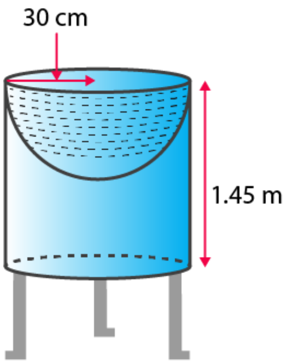
ANSWERS

Q.1	(2.5 cm)	Q.2	(1.2 cm)	Q.3	(0.1 cm)	Q.4	(8000)
Q.5	(0.05 cm)	Q.6	(3.85 Kg)	Q.7	(48 cm, 50 cm)	Q.8	(100π cm^3)
Q.9	($\frac{1}{4}$)	Q.10	345.39 g)				

4 Mark Questions

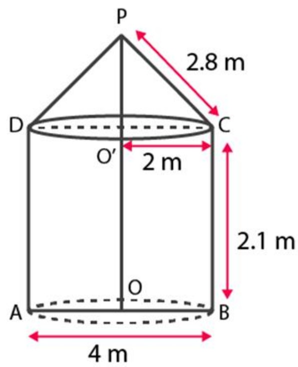
Q.1 Rameez got a playing top (Lattu) as his birthday gift which had no colour on it. He wanted to colour it with his crayons. The top is shaped like a cone surmounted by a hemisphere. The entire top is 5 cm in height and the diameter of the top is 3.5 cm. Find the area he has to colour. (take $\pi \frac{22}{7}$).

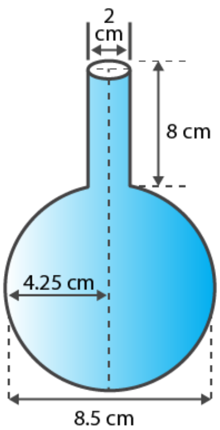




Q.2A person made a bird bath for his garden in the shape of a cylinder with a hemispherical depression at one end, as shown in the figure. The height of the cylinder is 1.45 m and its radius is 30 cm. Find the total surface area of the bird bath. (take $\pi \frac{22}{7}$).

Q.3 A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m. Find the area of the canvas used for making the tent.





Q.4 A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter, the diameter of the spherical part is 8.5 cm. By measuring the amount of water, it holds. Its volume is 345 cm3 found by a child. Check whether she is correct ($\pi = 3.14$)

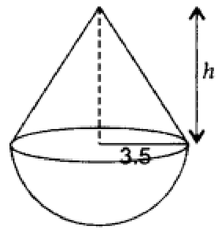
Q.5 A cone of height 24 cm and radius of base 6 cm is made up of modelling clay. A boy reshapes it in the form of a sphere. What is the radius of the sphere?

Q.6 Metallic spheres of radii 6 cm, 8 cm and 10 cm respectively are melted to form a single solid sphere. Find the radius of the resulting sphere.

Q.7 A canal is 300 cm wide, 120 cm deep. The water in the canal is flowing at a speed of 20 Km/h. How much area will it irrigate in 20 minutes, if 8 cm of standing water is desired.

Q.8 The radii of the circular ends of a bucket of height 15 cm are 14 cm and "r" cm ($r < 14$ cm). If the bucket has volume 5390 cm^3. Find "r". (Take $\pi \frac{22}{7}$).

Q.9 A solid wooden toy in the form of a hemisphere surmounted by a cone of same radius. The radius of the hemisphere is 3.5 cm and the total wood used in the toy is $\frac{1001}{6}$ cm^3. Find the height of the toy.



Q.10 A patient in a hospital is given soup daily in a cylindrical bowl of diameter 7 cm. If the soup is filled to a height of 4 cm. How much soup is to be given to 250 patients?

Q.11 A building is in the form of a cylinder surrounded by a hemispherical vaulted dome and contains $41\frac{19}{21}$ cum of air. If the internal diameter of the building is equal to the total height cu.m above the floor. Find the height of the building.

Q.12 A heap of sand forms a cone whose diameter is 10.5 m and height 3 m. Find its volume. If it is to be protected by canvas from rain. Find the area of the canvas required.

ANSWERS

Q.1	(39.6 cm^2) approx.	Q.2	(3.3 m^2)	Q.3	(44 m^2)	Q.5	(6 cm)
Q.6	(r = 12 cm)	Q.7	(300000 m^2)	Q.8	(r = 7 cm)	Q.9(h= 9.5 cm)	
Q.10	(38500 cm^3)	Q.11	(4 cm)	Q.12	(86.625 m^3 & 99.825 m^2)		