INDIAN ASSOCIATION OF PHYSICS TEACHERS

NATIONAL STANDARD EXAMINATION IN ASTRONOMY 2016 -17

Date of Examination: 27TH November, 2016

Time: 1330 to 1530 Hrs

Q. Paper Code: A421

Write the question paper code mentioned above on YOUR answer sheet (in the space provided), otherwise your answer sheet will NOT be assessed. Note that the same Q. P. Code appears on each page of the question paper.

Instructions to Candidates –

1. Use of mobile phones, smartphones, ipads during examination is **STRICTLY PROHIBITED**.
2. In addition to this question paper, you are given answer sheet along with Candidate's copy.
3. On the answer sheet, make all the entries carefully in the space provided **ONLY** in **BLOCK CAPITALS** as well as by properly darkening the appropriate bubbles.

Incomplete/ incorrect/carelessly filled information may disqualify your candidature.

4. On the answer sheet, use only **BLUE or BLACK BALL POINT PEN** for making entries and filling the bubbles.
5. The email ID and date of birth entered in the answer sheet will be your login credentials for accessing performance report. Please take care while entering.
6. Question paper has 80 multiple choice questions. Each question has four alternatives, out of which **only one** is correct. Choose the correct alternative and fill the appropriate bubble, as shown.

7. A correct answer carries 3 marks whereas 1 mark will be deducted for each wrong answer.
8. Any rough work should be done only in the space provided.
9. Use of **non-programmable** calculator is allowed.
10. No candidate should leave the examination hall before the completion of the examination.
11. After submitting your answer paper, take away the Candidate's copy for your reference.

Please DO NOT make any mark other than filling the appropriate bubbles properly in the space provided on the answer sheet.

Answer sheets are evaluated using machine, hence CHANGE OF ENTRY IS NOT ALLOWED.

Scratching or overwriting may result in a wrong score.

DO NOT WRITE ON THE BACK SIDE OF THE ANSWER SHEET.

Instructions to Candidates (continued) –

Read the following instructions after submitting the answer sheet.

12. **Comments regarding this question paper, if any, may be sent by email only to iapt.nse@gmail.com till 29th November, 2016.**
13. **The answers/solutions to this question paper will be available on our website – www.iapt.org.in by 2nd December, 2016.**
14. **CERTIFICATES and AWARDS –**

Following certificates are awarded by the IAPT to students successful in NSEs

(i)Certificates to "Centre Top 10%" students

(ii)Merit Certificates to "Statewise Top 1%" students

(iii)Merit Certificates and a book prize to "National Top 1%" students

15. Result sheets can be downloaded from our website in the month of February. The "Centre Top 10%" certificates will be dispatched to the Prof-in-charge of the centre by February, 2017.
16. List of students (with centre number and roll number only) having score above MAS will be displayed on our website (www.iapt.org.in) by **22nd December, 2016.** See the **Eligibility Clause** in the Student's brochure on our website.
17. Students eligible for the INO Examination on the basis of selection criteria mentioned in Student's brochure will be informed accordingly.

Indian Association of Physics Teachers

NATIONAL STANDARD EXAMINATION IN ASTRONOMY 2016-2017

Total time: 120 minutes **Marks: 240**

Only one out of four options is correct

1) Two identical stars with mass *M* orbit around their centre of mass in circular orbit. If radius of the orbit is *R* and the stars are always diametrically opposite. Consider the following statements

(i) Their binding force is equal to $\frac{GM^2}{4R^2}$

(ii) If the stars are heavier and closer, their orbital speed is greater.

(iii) The period of the orbit is $T = \pi\sqrt{\frac{R^3}{GM}}$

(iv) The minimum energy required to separate the two stars to infinity is equal to $\frac{GM^2}{4R}$

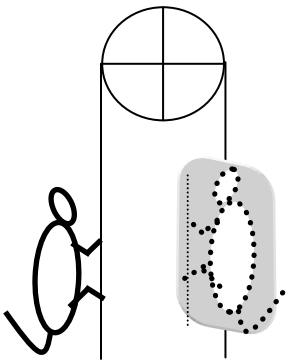
Select correct statement/s

a) Only (i) and (ii) b) Only (i), (ii) and (iv) c) Only (i), (iii) and (iv) d) Only (i) and (iii)

2) The number of natural numbers $n \leq 50$ such that $\sqrt[3]{n+\sqrt[3]{n+\sqrt[3]{n+\cdots}}}$ is a natural number is

a) Zero b) 2 c) 50 d) 5

3) A monkey is holding onto one end of a rope which passes over a frictionless pulley and at the other end is a plane mirror which has a mass equal to the mass of the monkey. At equilibrium the monkey is able to see her image in the mirror. How does the monkey see her image in the mirror as she climbs up the rope?



a) The image of the monkey moves with double speed of that of the monkey.
b) The image of the monkey moves with half the speed of that of the monkey.
c) The image of the monkey moves as fast as the monkey.
d) The monkey will not be able to see her image.

4) If $i = \sqrt{-1}$ then i^{2i} is a

a) purely imaginary number b) natural number
c) real number d) complex number with non-zero real and imaginary parts .

5) A steel ball is dropped from a height of 1 m on to a hard non-conducting surface. Every time it bounces it reaches 80 % of its previous height. All the losses in the energy are accounted only for increasing the temperature. Nearly how much is the rise in temperature of the ball just after the third bounce? (g = 10 m/s^2, specific heat capacity of material of the ball = 500 J/(kg·K))

a) 0.005 ^{0}C b) 0.01 ^{0}C c) 0.015 ^{0}C d) 0.02 ^{0}C

6) In a track event, a circumcircle and an incircle were drawn for a triangle having sides 50 m, 120 m and 130 m respectively. Ram and Sham were asked to walk on the in-circle and the circumcircle respectively. They started walking with same speed in the same direction (sense of rotation) from a point where they were closest. After how many rounds each, will they be closest again?

a) Ram 4 and Sham 13 b) Ram 13 and Sham 4
c) Ram 5, Sham 15 d) Ram 5, Sham 12

7) The angle between the two complex numbers $a = i^i$ and $b = 1$ is

a) π b) 0 c) $\frac{\pi}{2}$ d) $-\frac{\pi}{2}$

8) The number of rectangles that can be formed by joining the points of 4×4 grid of equispaced points is

a) 16 b) 36 c) 40 d) 42

9) A train of mass *m* is moving on a circular track of radius *r* with constant speed *v*. The Length of the train is exactly equal to half the circumference of the circular track. Magnitude of its linear momentum is

a) *mv/π* b) 0.5 *mv* c) *2mv/π* d) *mv*

10) The number of integers a, b, c for which $2a^2 + b^2 - 8c = 7$ is

a) 2 b) Infinite c) 0 d) 4

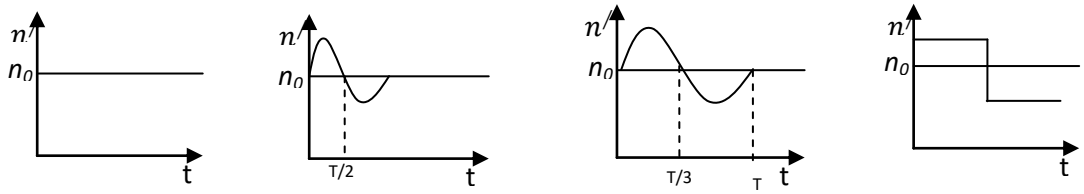
11) In SI units we use length, mass and time as fundamental quantities. Another intelligent world may not know these. However, three constants *G* (universal gravitational constant), *c* (speed of light in vacuum) and *h* (Planck's constant) are really universal and can be related to almost all the known interactions. In terms of these fundamental constants, the dimensions of time are

a) $\left[G^{\frac{1}{2}} c^{-\frac{5}{2}} h^{\frac{1}{2}}\right]$ b) $\left[G^{1} c^{-2} h^{\frac{1}{2}}\right]$ c) $\left[G^{2} c^{-\frac{1}{2}} h^{\frac{1}{2}}\right]$ d) $\left[G^{\frac{1}{2}} c^{-\frac{3}{2}} h^{\frac{1}{2}}\right]$

12) If $p(x) = x(x+1)(x+2)\ldots(x+2001) - c$ then the maximum multiplicity of the roots of $p(x)$ can be

a) 1 b) 2 c) 3 d) 2001

13) A train is running on a circular track of radius *R* with a constant speed. The driver is blowing siren of a constant frequency (n_0) throughout the circular motion of period *T*. There is a listener on the diameter of the track at a distance *R*/2 from centre of the circle. At *t* = 0, the train siren is farthest from the listener. In the following graphs the frequency, as recorded by the listener, is plotted against time. Which of them is closest to the correct pattern?

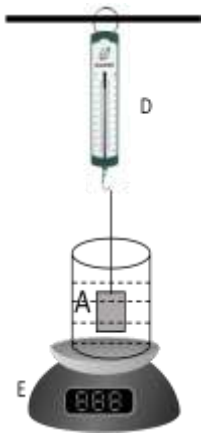


a) b) c) d)

14) If $\{x\}$ denotes the fractional part of a real number then $\int_0^{\sqrt{2}} \{x^2\} dx =$

a) $2\frac{\sqrt{2}}{3}$ b) $\frac{1}{3}$ c) $1 - \frac{\sqrt{2}}{3}$ d) $1 + \frac{\sqrt{2}}{3}$

15) Adjacent figure shows a block A, held by a spring balance D and submerged into a liquid in a beaker. The beaker is kept on a weighing balance E. Mass of the beaker plus the liquid is 2.5 kg. Balance D reads 2.5 kg and E reads 7.5 kg. Volume of the block is 0.003 m^3. Consider the following statements



(i) The density of the liquid is 5000/3 kg/m^3
(ii) The mass of block A is 7.5 kg
(iii) The buoyant force is 5 kg·wt.
(iv) If half the volume of the block is pulled out of the liquid, E would read 5 kg.

Select correct option(s).

a) (i), (iii) and (iv) b) (i), (ii) and (iv)

c) (i) and (iv) d) (i), (ii), (iii) and (iv)

16) AM-HM inequality for positive real numbers a, b, c states that $\frac{a+b+c}{3} \geq \frac{3abc}{ab+cb+ca}$. If a, b are positive irrational numbers then

a) $\frac{9ab}{2a+b} \leq a+b$ b) $\frac{9ab}{2a+b} \leq 1$ c) $\frac{9ab}{a+2b} \leq 2a+b$ d) $\frac{18ab}{2a+b} \leq a+2b$

17) The optical effects (phenomena) involved when we see a rainbow could be associated with

(i) internal reflection
(ii) dispersion
(iii) total internal reflection
(iv) deviation

select the correct options

a) (ii), (iii) and (iv) b) (i), (ii) and (iv) c) (i) and (iv) d) (iii) and (iv)

18) Which of the following statements are true about periodic functions defined on the set of real numbers
A: Sum of two functions with finite period is always a periodic function with finite period
B: The period of a function that is sum of two periodic functions with finite period is least common multiple of the period of two functions

a) A and B are correct b) A is correct but B is incorrect
c) A is false but B is correct d) A and B are false

19) Unaware about the fact that analog ammeters and voltmeters can also have zero error, a student recorded following readings while determining resistance by using Ohm's law.

Obs No.	Voltage/V	Current/mA	Obs No.	Voltage/V	Current/mA
1	1.0	40	4	7.0	160
2	3.0	80	5	9.0	200
3	5.0	120			

If the ammeter has no zero error, the zero error in the voltmeter is.

a) – 1 V b) - 1.5 V c) 0.5 V d) 1 V

20) The inverse function of the function $\sin x + \cos x$ is

a) $\sin^{-1} x + \cos^{-1} x$ b) $\frac{1}{\sin x + \cos x}$ c) $\sin^{-1}(\frac{x}{\sqrt{2}})$ d) $\sin^{-1}(\frac{x}{\sqrt{2}}) - \frac{\pi}{4}$

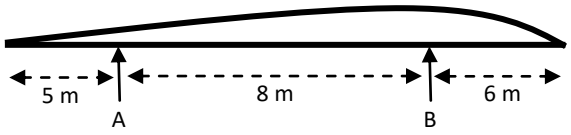
21) Particle A collides elastically (perfect) with another particle B which was at rest. They disperse in opposite directions with same speeds. Ratio of their masses must respectively be

a) 1:2 b) 1:3 c) 1:4 d) 2:3

22) $\lim_{x \to -\frac{\pi}{4}} \left(\frac{\sin x + \cos x}{x + \frac{\pi}{4}} \right) =$

a) ∞ b) $-\infty$ c) $\frac{1}{\sqrt{2}}$ d) $\sqrt{2}$

23) A long rowing boat put upside down, shown in the adjacent figure, has to be weighed using only a single bathroom scale. The boat will sag if it is supported only in the middle, and so the scales must be put first at position A with a wooden support at B, and then at position B with the wooden support at A. The readings on the scale are 45 kg and 55 kg respectively. The distance of centre of mass (from point A) and mass of the boat are respectively



a) 4.4 m, 120 kg b) 4.4 m, 100 kg c) 4.2 m, 100 kg d) 4.2 m, 120 kg

24) The number of real solutions of the equation $(x-1)(3^x - 2) = 1$ is

a) 0 b) 1 c) 2 d) More than 2

25) A car is fitted with a rear view mirror of focal length 20 cm. Another car, 2.8 m behind the first car is 15 m·s^{-1} faster than the first car and approaching. The relative speed of image of the second car, with respect to first car at this instant, is

a) 1/15 m·s^{-1} b) 1/10 m·s^{-1} c) 1/5 m·s^{-1} d) 2/15 m·s^{-1}

26) $\log_{\sqrt{2}} 16 + \log_{27} 9 + \log_{\frac{1}{3}} 3$

a) Is defined but cannot be found b) Is not defined
c) Is defined and equals $-\frac{1}{3}$ d) Is defined and equals $\frac{23}{3}$

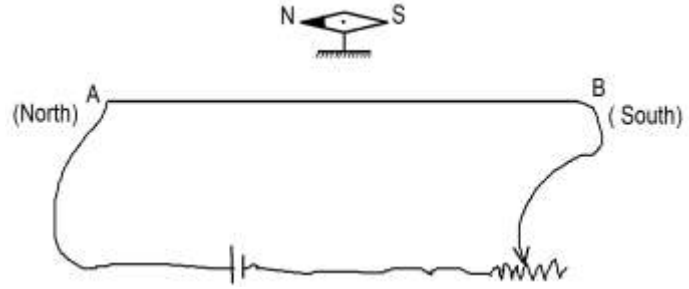
27) An electric buggy is stationed exactly midway between two plane vertical walls parallel to each other. A man standing adjacent to buggy blows whistle momentarily. Instantly the buggy starts running towards one of the walls with a velocity 35 m/s. The driver of the buggy records first two echoes of the whistle with a delay of exactly one second. Speed of sound in air at that temperature is 350 m/s. Distance between the walls must be

a) 433.125 m b) 866.25 m c) 1732.5 m d) 3465 m

28) The number of points at which $|x^3 - 1|$ is not differentiable is

a) 3 b) 2 c) 1 d) 0

29) The circuit given below has a long straight wire AB placed horizontal along North-South direction . A small magnetic needle can be held anywhere near this wire. Choose the correct statement.



a) North Pole of the magnetic needle will deflect towards East, if the compass is just above the wire.
b) North pole of the magnetic needle will deflect towards West, if the compass is at exactly same level of the wire.
c) North pole of the magnetic needle will deflect towards East, if the compass is just below the wire.
d) Magnetic needle will not deflect, if kept just below the wire.

30) If $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DA}$ are unit vectors such that $\overline{AB} \cdot \overline{BC} = \frac{1}{\sqrt{2}}$then

a) Points A, B, C, D are concyclic
b) Quadrilateral ABCD has area $\frac{1}{2\sqrt{2}}$
c) Quadrilateral ABCD has half of the maximal area for quadrilateral with same perimeter
d) The area determined by the vectors is $\frac{1}{\sqrt{2}}$

31) INSAT series of satellites are launched by India for telecommunication. Such satellites *appear stationary* at a particular point in the sky when observed from the earth. Consider the following statements:

(i) The satellite always experiences gravitation of the earth.
(ii) The satellite does not need any fuel for its motion.
(iii) The satellite does not experience net force.
(iv) Such satellites have to be positioned vertically above the equator.

a) Only (ii), (iii) & (iv) are correct.
b) Only (i), (ii) & (iv) are correct.
c) Only (i) & (iii) are correct.
d) Only (i) & (ii) are correct.

32) The number $3^8(3^{10} + 6^5) + 2^3(2^{12} + 6^7)$ is

a) A perfect square and a perfect cube
b) Neither a perfect square nor a perfect cube
c) A perfect cube but not a perfect square
d) A perfect square but not a perfect cube

33) Evaporation of (sweat) water is an essential mechanism in human beings for maintaining normal body temperature. For human beings, heat of vaporization of water at a body temperature of 37 ^{0}C is nearly 2.3 MJ/kg and specific heat capacity is 3.5 kJ/(kg·K). On consuming a certain prescribed diet, the body temperature of an athlete of mass 82 kg is expected to increase by 2 ^{0}C. In order to prevent this, he drinks N bottles of mineral water (250 ml water in each) at 37 ^{0}C. Assume that the entire amount of this water is given out as sweat, which vaporizes. N is nearly (density of water = 1000 kg·m^{-3})

a) 1
b) 2
c) 3
d) 4

34) The number $n = 1 + 12 + 60 + 160 + 240 + 192 + 64$ is

a) A perfect square and a perfect cube
b) Neither a perfect square nor a perfect cube
c) A perfect cube but not a perfect square
d) A perfect square but not a perfect cube

35) A football (assumed to be a hollow sphere) of mass 200 g and radius 16 cm, is given horizontal kick 4 cm above its centre. This imparts a speed of 8 cm/s to the football. Angular speed acquired by the football in radian/s is

a) 9/16 b) 15/16 c) 3/4 d) 16/15

36) For sets A, B we have (here X^c denote complement of set X) $(A \times B)^c =$

a) $A^c \times B^c$
b) $B^c \times A^c$
c) $A^c \times B \cup B^c \times A \cup A^c \times B^c$
d) $A^c \times B \cup A \times B^c \cup A^c \times B^c$

37) A small marble (assumed to be a uniform solid sphere) is released on one end of a parabolic mirror from a vertical height of 70 cm. First half part of this mirror is rough on which the marble is released. Other half of the mirror is smooth. Throughout its motion the marble never slips. To what vertical height will it rise on the smooth surface?

a) 98 cm b) 70 cm c) 63 cm d) 50 cm

38) The number $3^{12} + 2^9 + 3(3 \times 6^4 + 6^5) + 2^6$ is

a) A perfect square and a perfect cube
b) A perfect cube but not a perfect square
c) A perfect square but not a perfect cube
d) Neither a perfect square nor a perfect cube

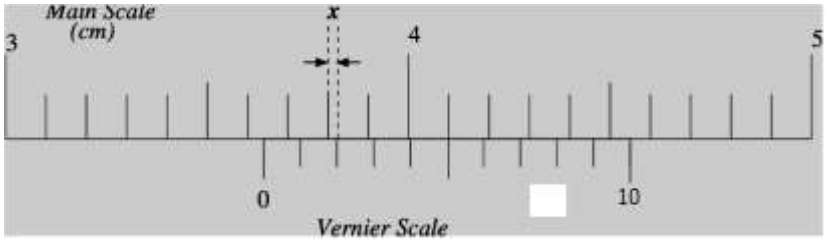
39) Radius and moment of inertia of a smooth pulley are 0.1 m and 1 kg·m^2 respectively. A tangential force of $f = 40t - 10t^2$ sets the pulley into rotation. Direction of its rotation reverses after some time. The time duration after which the direction will reverse is

a) 6 s b) 8 s c) 4 s d) 12 s

40) $\log_{10} 0.01 + \log_{0.1} 10 + \log_{10} 0.001 + \log_{0.1} 0.001 =$

a) $\log_{10.2} 10.012$ b) $\log_{10} 0.000001 + 3$ c) $-4 + \log_2 8$ d) None of the above

41) The figure shows a particular position of a Vernier callipers. The value of *x* in cm is



a) 0.03 b) 0.15 c) 3.83 d) 0.02

42) If A and B are two sets then the set $A \times B$ is given by

a) $\{a \times b | a \in A, b \in B\}$ b) $\{(a,b) | a \in B, b \in A\}$
c) $\{(a,b) | a \in A, b \in B\}$ d) $\{ab | a \in A,\ b \in B\}$

43) A lens is held directly above a pencil lying on a floor and forms an image of it. On moving the lens vertically through a distance equal to its focal length, it again forms image of same size as that of the previous image. If the length of the pencil is 5.0 cm, the length of the image is

a) 10.0 cm b) 15.0 cm c) 20.0 cm d) 12.5 cm

44) If $A = \{2,3\}, B = \{4,5\}$ then $A \times B =$

a) $\{8,15\}$ b) $\{8,10\ 12,15\}$ c) $\{(2,4),(3,5)\}$ d) $\{(2,4),(2,5),(3,4)(3,5)\}$

45) A balloon *less dense than air* is tied at the floor of a truck with a massless, inextensible and flexible string. The truck is observed to be taking a left turn. Select correct statement.

a) The string will incline towards right (outward, w.r.t. person in the truck).
b) The string will incline towards left (inward, w.r.t. person in the truck).
c) The string will still be vertical as the balloon is less dense than air.
d) Buoyant force on the balloon is equal to weight of the balloon.

46) A function F from A to B is

a) Relation F with $(a,b),(c,b) \in F \Rightarrow a = c$ b) $F \subset A \times B$
c) Relation F with $(a,b),(a,c) \in F \Rightarrow b = c$ d) Relation F with $(a,b),(b,c) \in F \Rightarrow (a,c) \in F$

47) An ice cube with a steel ball bearing trapped inside it is floating above water in a glass. What will happen to the water level in the glass after the ice melts completely?

a) Rise b) Fall
c) will not change d) Answer depends upon actual position of the steel ball.

48) Which of the following is a mathematically acceptable statement?

a) It is an even number b) 13th December is Saturday
c) Common donkey belongs to class orthopoda d) Alexander was a great king

49) A block of mass 5 kg is to be dragged along a rough horizontal surface having μ_s = 0.5 and μ_k = 0.3. The horizontal force applied for dragging it is 20 N. (g = 10 m/s^2). Select correct statement/s.

a) Frictional force acting on the block is 20 N. b) Block will be displaced.
c) Block will move with acceleration 1 m/s^2. d) Block will initially move and then stop.

50) The negation of the statement: $'f(x)$ is continuous for all real numbers $x.'$ is

a) $f(x)$ is not continuous for all real numbers x
b) $f(x)$ is not continuous for any real number x
c) $f(x)$ is not continuous for every real number x
d) $f(x)$ is not continuous for some real number x

51) A bullet moving with a speed of 72 m/s comes to a halt in a fixed wooden block on travelling 9 cm inside it. If the wooden block (of the same type of wood) were to be 8 cm thick, the bullet would come out of the block with a speed.

a) 9 $m \cdot s^{-1}$ b) 8 $m \cdot s^{-1}$ c) 24 $m \cdot s^{-1}$ d) 64 $m \cdot s^{-1}$

52) Let l be a vertical line and m a line that makes an angle of $\frac{\pi}{6}$ with l. Consider the cone generated by rotating m around the axis l. If plane L makes an angle of 15^0 with line l then the intersection of the plane and the cone is

a) A Parabola b) A pair of straight lines c) An ellipse d) A hyperbola

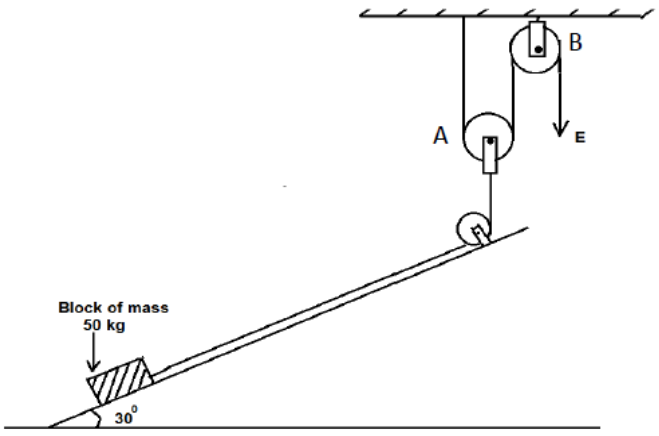
53) A piece of brass (an alloy of copper and zinc) weighs 12.9 g in air. When completely immersed in water, it weighs 11.3 g. What is the mass of copper contained in the alloy? The density of copper and zinc are 8.9 g/cm^3 and 7.1 g/cm^3 respectively.

a) 6.89 g b) 4.54 g c) 8.93 g d) 7.61 g

54) The coefficients of x in the expansion of $(1+x)^5$ correspond to the

a) 5^{th} row of Pascal's triangle b) 6^{th} row of Pascal's triangle
c) 7^{th} row of Pascal's triangle d) 4^{th} row of the Pascal's triangle

55) **Linked questions (55-56):** The adjacent figure shows a ramp (30^0) holding a block of mass 50 kg. The block attached to a movable pulley *A* with an inextensible massless string. The movable pulley is in turn held with the help of another fixed pulley *B*. The block kept on the ramp is to be raised through a ***vertical*** height of 10 cm. By what distance the string should be lowered down vertically, below *E*?



a) 20 cm b) 5 cm

c) 40 cm d) $10(3^{1/2})$ cm

56) Refer to figure in question 55. Pulleys in the figure are massless and frictionless. Neglecting friction between block and the ramp, the force that should be applied vertically downwards, at *E*, to slide the block along the ramp without acceleration is (g=10 $m \cdot s^{-1}$)

a) 65 N b) 125 N c) 175 N d) 250 N

57) If $\cos^2 x = \frac{1}{3}$ then $cosec\ x =$

a) $\sqrt{3}$ b) $\frac{2}{\sqrt{3}}$ c) $\sqrt{\frac{2}{3}}$ d) $\sqrt{\frac{3}{2}}$

58) Which of digits 1, 3, 4, 5, 7, 0 **cannot** appear at the ten's place of powers of 3?

a) 1, 3, 5 only b) 1, 3, 4, 7 only c) 1, 3, 5, 7 only d) 0, 1, 3, 5 only

59) A fish is swimming in still water. At a given instant there is a bird flying vertically above the fish. For the bird the fish appears to be 15 m below the surface of water while for the fish the bird appears to be 20 m above the surface. Refractive index of water is 4/3. Actual distance between the bird and the fish is

a) 35 m b) 40 m c) 30 m d)25 m

60) The smallest integer n such that $\sqrt{n+1}-\sqrt{n} \leq 0.01$ is

a) 2499 b) 2500 c) 2501 d) 2502

61) A uniform wire of resistance per unit length 1 Ω/m is bent in the form of an equilateral triangle. If effective resistance between adjacent vertices is 2.4 Ω, length of each side of the triangle is

a) 1.8 m b) 2.4 m c) 3.6 m d) 7.2 m

62) ΔABC is equilateral with each side being of unit length and P is an interior point, then the maximum product of the lengths AP, BP and CP is

a) $\frac{1}{5\sqrt{3}}$ b) $\frac{1}{4\sqrt{3}}$ c) $\frac{1}{3\sqrt{3}}$ d) $\frac{1}{6}$

63) The resultant of the forces P and Q is *R* If Q is doubled then *R* gets doubled. If Q is reversed, even then *R* gets doubled. Then

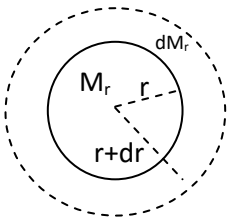
a) $P: Q: R = \sqrt{2}: \sqrt{3}: \sqrt{2}$ b) $P: Q: R = \sqrt{2}: \sqrt{2}: \sqrt{3}$
c) $P: Q: R = \sqrt{3}: \sqrt{3}: \sqrt{2}$ d) $P: Q: R = \sqrt{2}: \sqrt{3}: \sqrt{3}$

64) The unit's digit of $23^{2015} \times 7^{2016} \times 13^{2017}$ is

a) 1 b) 3 c) 7 d) 9

65) **Linked questions (65-69)**

A star can be considered as a spherical ball of hot gas of radius R. Inside the star, the density of the gas is ρ_r at a radius *r* and mass of the gas within this region is M_r. The correct differential equation for variation of mass with respect to radius is (refer to the adjacent figure)



a) $\frac{dM_r}{dr} = \frac{4}{3}\pi\rho_r r^3$ b) $\frac{dM_r}{dr} = 4\pi\rho_r r^2$
c) $\frac{dM_r}{dr} = \frac{2}{3}\pi\rho_r r^2$ d) $\frac{dM_r}{dr} = \frac{1}{3}\pi\rho_r r^3$

66) A star in its prime age is said to be under equilibrium due to gravitational pull and outward radiation pressure (*P*). Consider the shell of thickness *dr* as in the figure of question (65). If the pressure on this shell id *dP* then the correct equation is (G is universal gravitational constant)

a) $\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho_r$ b) $\frac{dP}{dr} = \frac{GM_r}{r^2}\rho_r$ c) $\frac{dP}{dr} = -\frac{2}{3}\frac{GM_r}{r^2}\rho_r$ d) $\frac{dP}{dr} = \frac{2}{3}\frac{GM_r}{r^2}\rho_r$

67) In astronomy, order of magnitude estimation plays an important role. The derivative $\frac{dP}{dr}$ can be taken as difference ratio $\frac{\Delta P}{\Delta r}$. Consider the star has a radius R, pressure at its centre is P_c and pressure at outer most layer is zero. If the average mass is $\frac{M_0}{2}$ and average radius $\frac{R_0}{2}$ then the expression for P_c is

a) $P_c = \frac{3}{2}\frac{GM_0^2}{\pi R_0^4}$ b) $P_c = \frac{3}{4}\frac{GM_0^2}{\pi R_0^4}$ c) $P_c = \frac{2}{3}\frac{GM_0^2}{\pi R_0^4}$ d) $P_c = \frac{3}{2}\frac{GM_0^2}{R_0^4}$

68) The value of mass and radius of sun are given by $M_0 = 2 \times 10^{30}$ kg and $R_0 = 7 \times 10^5$ km respectively. The pressure at the centre is about (G = 6.67 × 10^{-11} $m^3 \cdot kg^{-1} \cdot s^{-2}$)

a) $2x10^{14}\ N\cdot m^{-2}$ b) $2\ x10^{15}\ N\cdot m^{-2}$ c) $5x10^{14}\ N\cdot m^{-2}$ d) $7x10^{15}\ N\cdot m^{-2}$

69) Assuming that the gas inside the sun behaves very much like the perfect gas, the temperature at the centre of the sun is nearly (the number density of gas particles = $\frac{2\rho}{M_H}$, Boltzmann constant $k_B = 1.4 \times 10^{-23}\ J\cdot K^{-1}$ and mass of proton $M_H = 1.67 \times 10^{-27}$ kg)

a) $3\ X\ 10^7$ K b) $2\ X10^7$ K c) $4X10^7$ K d) $6X10^7$ K

70) At the earth's equator a satellite is observed passing directly overhead moving west to east in the sky. Exactly 12 hours later, satellite is again observed directly overhead. The altitude of the satellite is (Radius of the earth = 6400 km)

a) $1.82X10^7$ m b) $1.39\ X10^7$ m c)$3.59\ X10^7$ m d) $6.4\ X10^7$ m

71) **Passage 71 to 73**

Two stars, with masses M_1 and M_2 are in circular orbits around their common centre of mass. The star with mass M_1 has an orbit of radius R_1 and the star with mass M_2 has an orbit of radius R_2. The correct relation is

a) $\frac{R_1}{R_2} = \frac{M_2}{M_1}$ b) $\frac{R_1}{R_2} = \frac{M_1}{M_2}$ c) $\frac{R_1}{R_2} = \sqrt{\frac{M_2}{M_1}}$ d) $\frac{R_1}{R_2} = \sqrt{\frac{M_1}{M_2}}$

72) The time period of each of the star is

a) $T^2 = \frac{4\pi^2(R_1+R_2)^2R_2}{GM_2}$ b) $T^2 = \frac{4\pi^2(R_1+R_2)^3}{G((M_1+M_2)}$

c) $T^2 = \frac{4\pi^2(R_1+R_2)^2R_2}{GM_1}$ d) $T^2 = \frac{4\pi^2(R_1+R_2)^2R_1}{GM_1}$

73) The two stars in certain binary system move in circular orbits. The first star alpha has an orbital speed of 36.0 $km\cdot s^{-1}$. The second star, beta has an orbital speed of 12.0 $km\cdot s^{-1}$. The orbital period of first star is 137 days. The mass of the two stars are about

a)$2.1 \times 10^{30}\ and\ 6.8 \times 10^{30}$ kg b) $1.3 \times 10^{30}\ and\ 3.9 \times 10^{30}$ kg

c)$3.5 \times 10^{30}\ and\ 9.2 \times 10^{30}$ kg d) $0.8 \times 10^{30}\ and\ 2.3 \times 10^{30}$ kg

74) **Passage question 74-76**

Consider a spacecraft in an elliptical orbit around the earth. At the low point or perigee of its orbit, it is 400 km above the earth's surface. At the high point or apogee it is 4000 km above the earth's surface. The period of the space craft's orbit is (g=9.8ms^{-2} and R=6400 km)

a) 0.29 hr b) 1.82 hrs c) 2.21 hrs d) 3.56 hrs

75) The ratio of speed of the spacecraft at perigee to its speed at apogee is almost equal to

a) 10:1 b) 3:2 c) 2:3 d) 1:10

76) The speed of the satellite at perigee is

a) 8576 $m \cdot s^{-1}$ b) 57,307 $m \cdot s^{-1}$ c) 5876 $m \cdot s^{-1}$ d) 7856 $m \cdot s^{-1}$

77) Astronomers believe that a large percentage of the mass of the universe is dark matter. In one recent study, the transverse velocity of the Large Magellanic cloud (LMC) was measured to be 200 $km \cdot s^{-1}$. The LMC is believed to orbit the centre of our galaxy at about 17 $X10^4$ ly ($1.6X10^{21}$ m). Assuming a circular orbit, percentage of dark matter in our galaxy is about (Independent estimate of visible matter is $2X10^{41}$ kg)

a) 77 % b) 82 % c) 70 % d) 80 %

78) The escape speed from Jupiter is approximately 59.5 $km \cdot s^{-1}$ and its radius is about 12 times that of earth. From this we may estimate the mean density of Jupiter to be about (Radius of earth = 6400 km and escape speed from the earth is 11.2 $km \cdot s^{-1}$)

a) 5 times that of earth b) 0.2 times that of the earth
c) 2.5 times that of the earth d) 0.4 times that of the earth

79) The orbit of planet mercury has the largest eccentricity of about 0.2 in the solar system. If the maximum distance of mercury from the centre of the sun is about 69 million Km, its minimum distance from sun is about

a) 13.8 million km b) 57.5 million km c) 46 million km d) 18 million km

80) As observed from a place in Australia the pole star

a) appears in the southern direction b) appears at about 30^0 above the horizon
c) much brighter than that seen from India d) can never be seen

Rough Sheet

Rough Sheet

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