

1

Similarity



Let's study.

- Ratio of areas of two triangles
- Basic proportionality theorem
- Converse of basic proportionality theorem
- Tests of similarity of triangles
- Property of an angle bisector of a triangle
- Property of areas of similar triangles
- The ratio of the intercepts made on the transversals by three parallel lines



Let's recall.

We have studied Ratio and Proportion. The statement, 'the numbers a and b are in the ratio $\frac{m}{n}$ ' is also written as, 'the numbers a and b are in proportion $m:n$.'

For this concept we consider positive real numbers. We know that the lengths of line segments and area of any figure are positive real numbers.

We know the formula of area of a triangle.

$$\text{Area of a triangle} = \frac{1}{2} \text{ Base} \times \text{Height}$$



Let's learn.

Ratio of areas of two triangles

Let's find the ratio of areas of any two triangles.

Ex. In $\triangle ABC$, AD is the height and BC is the base.

In $\triangle PQR$, PS is the height and QR is the base

$$\frac{A(\triangle ABC)}{A(\triangle PQR)} = \frac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times QR \times PS}$$

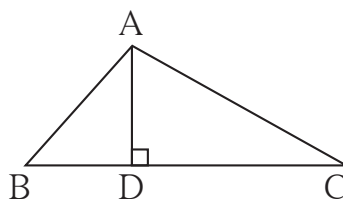


Fig. 1.1

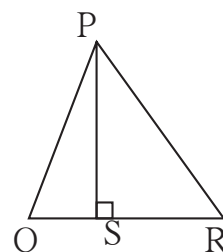


Fig. 1.2

$$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{BC \times AD}{QR \times PS}$$

Hence the ratio of the areas of two triangles is equal to the ratio of the products of their bases and corresponding heights.

Base of a triangle is b_1 and height is h_1 . Base of another triangle is b_2 and height is h_2 . Then the ratio of their areas = $\frac{b_1 \times h_1}{b_2 \times h_2}$

Suppose some conditions are imposed on these two triangles,

Condition 1: If the heights of both triangles are equal then-

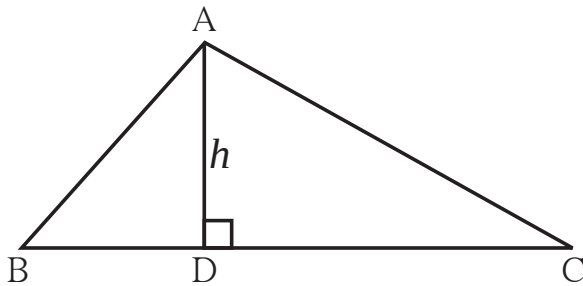


Fig. 1.3

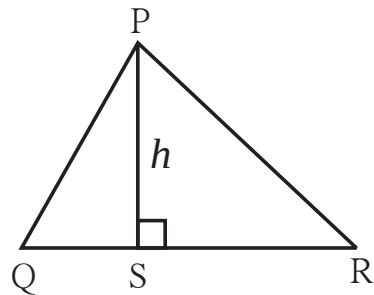


Fig. 1.4

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{BC \times h}{QR \times h} = \frac{BC}{QR}$$

$$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{b_1}{b_2}$$

Property: The ratio of the areas of two triangles with equal heights is equal to the ratio of their corresponding bases.

Condition 2: If the bases of both triangles are equal then -

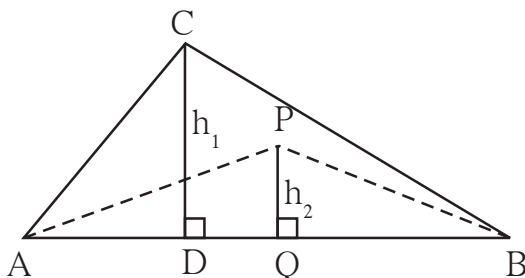


Fig. 1.5

$$\frac{A(\Delta ABC)}{A(\Delta APB)} = \frac{AB \times h_1}{AB \times h_2}$$

$$\frac{A(\Delta ABC)}{A(\Delta APB)} = \frac{h_1}{h_2}$$

Property: The ratio of the areas of two triangles with equal bases is equal to the ratio of their corresponding heights.

Activity :

Fill in the blanks properly.

(i)

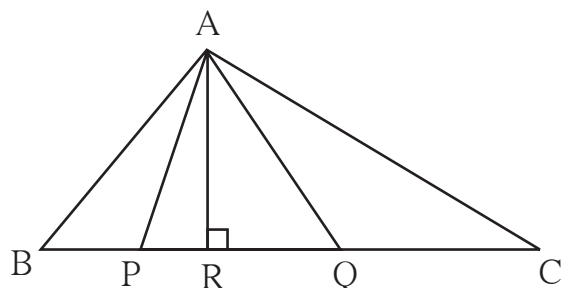


Fig. 1.6

$$\frac{A(\triangle ABC)}{A(\triangle APQ)} = \frac{\boxed{} \times \boxed{}}{\boxed{} \times \boxed{}} = \frac{\boxed{}}{\boxed{}}$$

(ii)

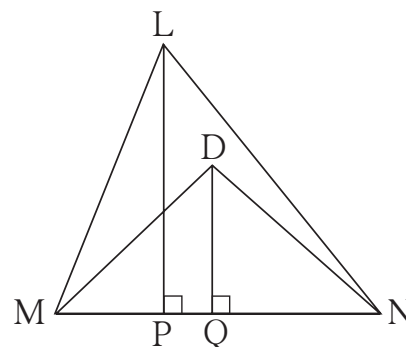


Fig.1.7

$$\frac{A(\triangle LMN)}{A(\triangle DMN)} = \frac{\boxed{} \times \boxed{}}{\boxed{} \times \boxed{}} = \frac{\boxed{}}{\boxed{}}$$

(iii)

M is the midpoint of
seg AB and seg CM is a median
of $\triangle ABC$

$$\begin{aligned} \therefore \frac{A(\triangle AMC)}{A(\triangle BMC)} &= \frac{\boxed{}}{\boxed{}} \\ &= \frac{\boxed{}}{\boxed{}} = \boxed{} \end{aligned}$$

State the reason.

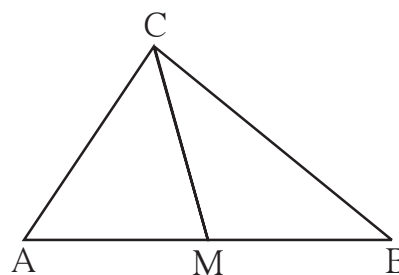


Fig. 1.8

~~~~~ Solved Examples ~~~~~

Ex. (1)

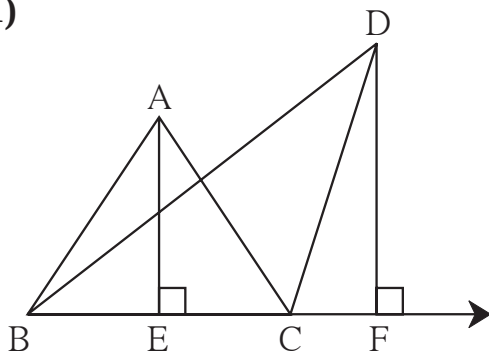


Fig.1.9

In adjoining figure

$AE \perp \text{seg } BC$, $\text{seg } DF \perp \text{line } BC$,

$AE = 4$, $DF = 6$, then find $\frac{A(\triangle ABC)}{A(\triangle DBC)}$.

Solution : $\frac{A(\triangle ABC)}{A(\triangle DBC)} = \frac{AE}{DF}$ bases are equal, hence areas proportional to heights.

$$= \frac{4}{6} = \frac{2}{3}$$

Ex. (2) In $\triangle ABC$ point D on side BC is such that $DC = 6$, $BC = 15$. Find $A(\triangle ABD) : A(\triangle ABC)$ and $A(\triangle ABD) : A(\triangle ADC)$.

Solution : Point A is common vertex of $\triangle ABD$, $\triangle ADC$ and $\triangle ABC$ and their bases are collinear. Hence, heights of these three triangles are equal

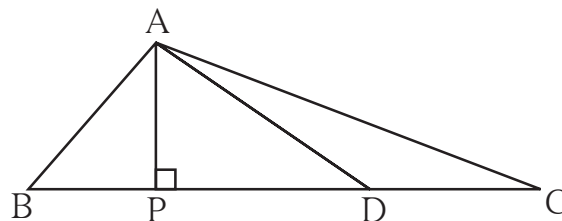


Fig. 1.10

$$BC = 15, DC = 6 \therefore BD = BC - DC = 15 - 6 = 9$$

$$\frac{A(\triangle ABD)}{A(\triangle ABC)} = \frac{BD}{BC} \dots\dots\dots \text{heights equal, hence areas proportional to bases.}$$

$$= \frac{9}{15} = \frac{3}{5}$$

$$\frac{A(\triangle ABD)}{A(\triangle ADC)} = \frac{BD}{DC} \dots\dots\dots \text{heights equal, hence areas proportional to bases.}$$

$$= \frac{9}{6} = \frac{3}{2}$$

Ex. (3)

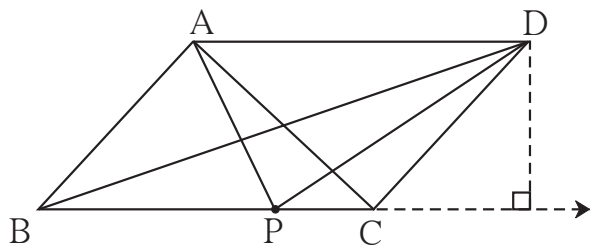


Fig. 1.11

□ ABCD is a parallelogram. P is any point on side BC. Find two pairs of triangles with equal areas.

Solution : □ ABCD is a parallelogram.

$$\therefore AD \parallel BC \text{ and } AB \parallel DC$$

Consider $\triangle ABC$ and $\triangle BDC$.

Both the triangles are drawn in two parallel lines. Hence the distance between the two parallel lines is the height of both triangles.

In $\triangle ABC$ and $\triangle BDC$, common base is BC and heights are equal.

$$\text{Hence, } A(\triangle ABC) = A(\triangle BDC)$$

In $\triangle ABC$ and $\triangle ABD$, AB is common base and heights are equal.

$$\therefore A(\triangle ABC) = A(\triangle ABD)$$

Ex.(4)

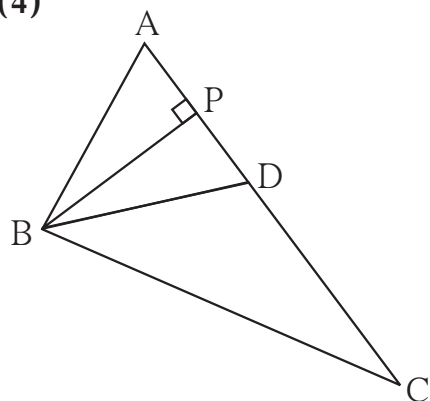


Fig. 1.12

In adjoining figure in $\triangle ABC$, point D is on side AC. If $AC = 16$, $DC = 9$ and $BP \perp AC$, then find the following ratios.

- (i) $\frac{A(\triangle ABD)}{A(\triangle ABC)}$ (ii) $\frac{A(\triangle BDC)}{A(\triangle ABC)}$
- (iii) $\frac{A(\triangle ABD)}{A(\triangle BDC)}$

Solution : In $\triangle ABC$ point P and D are on side AC, hence B is common vertex of $\triangle ABD$, $\triangle BDC$, $\triangle ABC$ and $\triangle APB$ and their sides AD, DC, AC and AP are collinear. Heights of all the triangles are equal. Hence, areas of these triangles are proportional to their bases. $AC = 16$, $DC = 9$

$$\therefore AD = 16 - 9 = 7$$

$$\therefore \frac{A(\triangle ABD)}{A(\triangle ABC)} = \frac{AD}{AC} = \frac{7}{16} \dots\dots\dots \text{triangles having equal heights}$$

$$\frac{A(\triangle BDC)}{A(\triangle ABC)} = \frac{DC}{AC} = \frac{9}{16} \dots\dots\dots \text{triangles having equal heights}$$

$$\frac{A(\triangle ABD)}{A(\triangle BDC)} = \frac{AD}{DC} = \frac{7}{9} \dots\dots\dots \text{triangles having equal heights}$$



Remember this!

- Ratio of areas of two triangles is equal to the ratio of the products of their bases and corresponding heights.
- Areas of triangles with equal heights are proportional to their corresponding bases.
- Areas of triangles with equal bases are proportional to their corresponding heights.



Practice set 1.1



1. Base of a triangle is 9 and height is 5. Base of another triangle is 10 and height is 6. Find the ratio of areas of these triangles.



2. In figure 1.13 $BC \perp AB$, $AD \perp AB$,
 $BC = 4$, $AD = 8$, then find $\frac{A(\triangle ABC)}{A(\triangle ADB)}$.

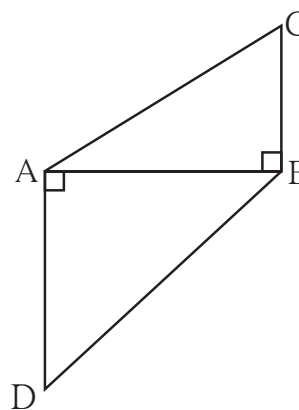


Fig. 1.13

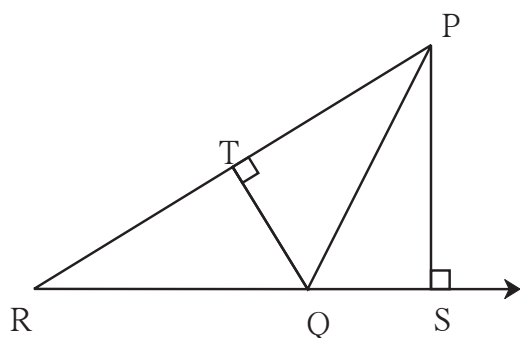


Fig. 1.14

4. In adjoining figure, $AP \perp BC$,
 $AD \parallel BC$, then find
 $A(\triangle ABC) : A(\triangle BCD)$.

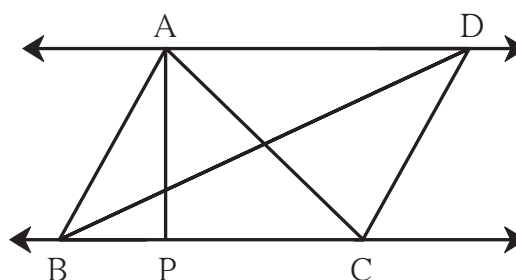


Fig. 1.15

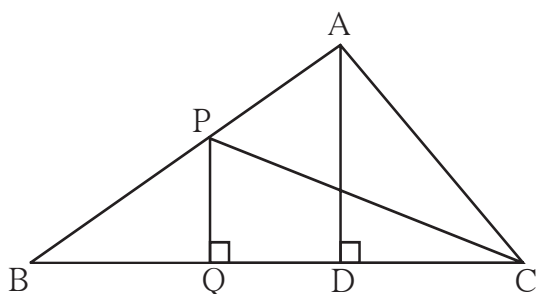


Fig. 1.16

5. In adjoining figure $PQ \perp BC$,
 $AD \perp BC$ then find following ratios.

- (i) $\frac{A(\triangle PQB)}{A(\triangle PBC)}$ (ii) $\frac{A(\triangle PBC)}{A(\triangle ABC)}$
 (iii) $\frac{A(\triangle ABC)}{A(\triangle ADC)}$ (iv) $\frac{A(\triangle ADC)}{A(\triangle PQC)}$



Let's learn.

Basic proportionality theorem

Theorem : If a line parallel to a side of a triangle intersects the remaining sides in two distinct points, then the line divides the sides in the same proportion.

Given : In ΔABC line $l \parallel$ line BC
and line l intersects AB and AC in point P and Q respectively

To prove : $\frac{AP}{PB} = \frac{AQ}{QC}$

Construction: Draw seg PC and seg BQ

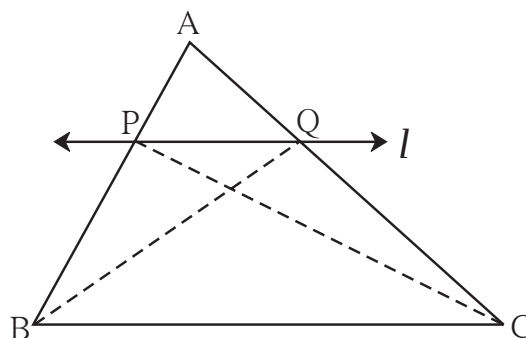


Fig. 1.17

Proof : ΔAPQ and ΔPQB have equal heights.

$$\therefore \frac{A(\Delta APQ)}{A(\Delta PQB)} = \frac{AP}{PB} \quad \dots\dots\dots \text{(I) (areas proportionate to bases)}$$

$$\text{and } \frac{A(\Delta APQ)}{A(\Delta PQC)} = \frac{AQ}{QC} \quad \dots\dots\dots \text{(II) (areas proportionate to bases)}$$

seg PQ is common base of ΔPQB and ΔPQC . seg $PQ \parallel$ seg BC ,
hence ΔPQB and ΔPQC have equal heights.

$$A(\Delta PQB) = A(\Delta PQC) \quad \dots\dots\dots \text{(III)}$$

$$\frac{A(\Delta APQ)}{A(\Delta PQB)} = \frac{A(\Delta APQ)}{A(\Delta PQC)} \quad \dots\dots\dots \text{[from (I), (II) and (III)]}$$

$$\therefore \frac{AP}{PB} = \frac{AQ}{QC} \quad \dots\dots\dots \text{[from (I) and (II)]}$$

Converse of basic proportionality theorem

Theorem : If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

In figure 1.18, line l intersects the side AB and side AC of ΔABC in the points P and Q respectively and $\frac{AP}{PB} = \frac{AQ}{QC}$, hence line $l \parallel$ seg BC .

This theorem can be proved by indirect method.

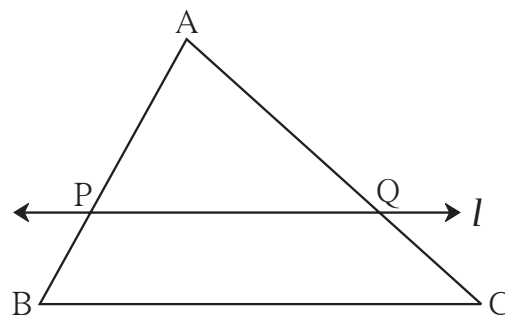


Fig. 1.18

Activity :

- Draw a $\triangle ABC$.
- Bisect $\angle B$ and name the point of intersection of AC and the angle bisector as D.
- Measure the sides.

AB = cm BC = cm

AD = cm DC = cm

- Find ratios $\frac{AB}{BC}$ and $\frac{AD}{DC}$.
- You will find that both the ratios are almost equal.
- Bisect remaining angles of the triangle and find the ratios as above. You can verify that the ratios are equal.

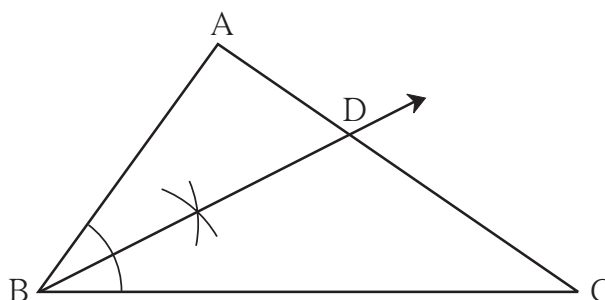


Fig. 1.19



Let's learn.

Property of an angle bisector of a triangle

Theorem : The bisector of an angle of a triangle divides the side opposite to the angle in the ratio of the remaining sides.

Given : In $\triangle ABC$, bisector of $\angle C$ intersects seg AB in the point E.

To prove : $\frac{AE}{EB} = \frac{CA}{CB}$

Construction : Draw a line parallel to ray CE, passing through the point B. Extend AC so as to intersect it at point D.

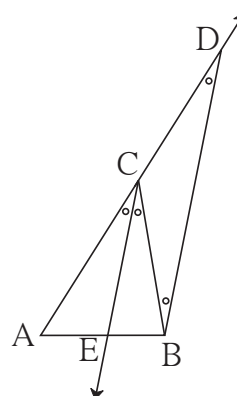


Fig. 1.20

Proof : ray $CE \parallel$ ray BD and AD is transversal,

$$\therefore \angle ACE = \angle CDB \quad \dots\dots\dots \text{(corresponding angles)} \dots\text{(I)}$$

Now taking BC as transversal

$$\angle ECB = \angle CBD \quad \dots\dots\dots \text{(alternate angle)} \quad \dots\text{(II)}$$

$$\text{But } \angle ACE \cong \angle ECB \quad \dots\dots\dots \text{(given)} \quad \dots\text{(III)}$$

$$\therefore \angle CBD \cong \angle CDB \quad \dots\dots\dots [\text{from (I), (II) and (III)}]$$

In $\triangle CBD$, side $CB \cong$ side CD (sides opposite to congruent angles)

$$\therefore CB = CD \quad \dots\text{(IV)}$$

Now in $\triangle ABD$, seg $EC \parallel$ seg BD (construction)

$$\therefore \frac{AE}{EB} = \frac{AC}{CD} \quad \dots\dots\text{(Basic proportionality theorem)}\dots\text{(V)}$$

$$\therefore \frac{AE}{EB} = \frac{AC}{CB} \quad \dots\dots\dots [\text{from (IV) and (V)}]$$

For more information :

Write another proof of the theorem yourself.

Draw $DM \perp AB$ and $DN \perp AC$. Use the following properties and write the proof.

- (1) The areas of two triangles of equal heights are proportional to their bases.

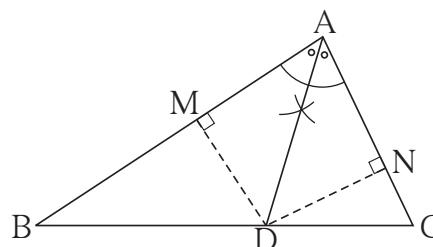


Fig. 1.21

- (2) Every point on the bisector of an angle is equidistant from the sides of the angle.

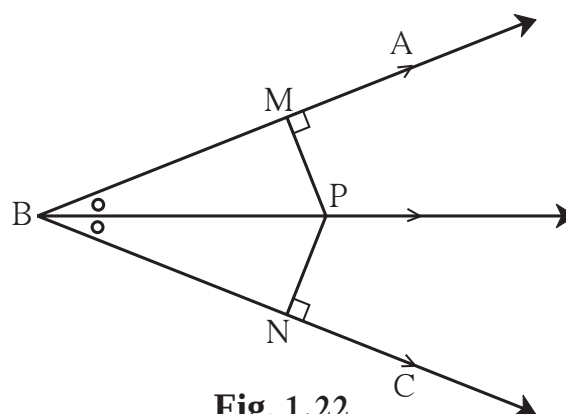


Fig. 1.22

Converse of angle bisector theorem

If in ΔABC , point D on side BC such that $\frac{AB}{AC} = \frac{BD}{DC}$, then ray AD bisects $\angle BAC$.

Property of three parallel lines and their transversals

Activity:

- Draw three parallel lines.
- Label them as l, m, n .
- Draw transversals t_1 and t_2 .
- AB and BC are intercepts on transversal t_1 .
- PQ and QR are intercepts on transversal t_2 .

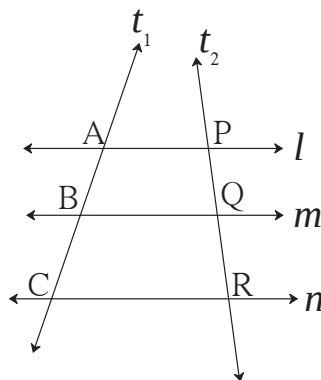


Fig. 1.23

- Find ratios $\frac{AB}{BC}$ and $\frac{PQ}{QR}$. You will find that they are almost equal.

Theorem : The ratio of the intercepts made on a transversal by three parallel lines is equal to the ratio of the corresponding intercepts made on any other transversal by the same parallel lines.

Given : line $l \parallel$ line $m \parallel$ line n

t_1 and t_2 are transversals.

Transversal t_1 intersects the lines in points A, B, C and t_2 intersects the lines in points P, Q, R.

To prove : $\frac{AB}{BC} = \frac{PQ}{QR}$

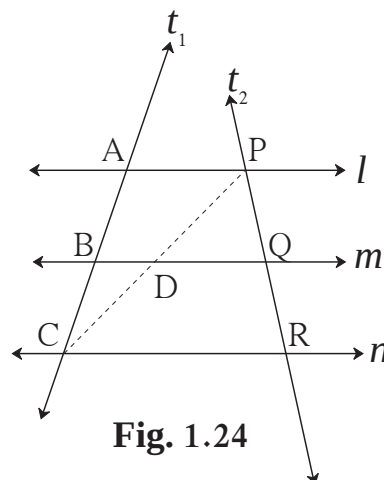


Fig. 1.24

Proof : Draw seg PC, which intersects line m at point D.

In ΔACP , $BD \parallel AP$

$$\therefore \frac{AB}{BC} = \frac{PD}{DC} \dots \dots (I) \text{ (Basic proportionality theorem)}$$

In ΔCPR , $DQ \parallel CR$

$$\therefore \frac{PD}{DC} = \frac{PQ}{QR} \dots \dots (II) \text{ (Basic proportionality theorem)}$$

$$\therefore \frac{AB}{BC} = \frac{PD}{DC} = \frac{PQ}{QR} \dots \dots \text{from (I) and (II).} \qquad \therefore \frac{AB}{BC} = \frac{PQ}{QR}$$





Remember this!

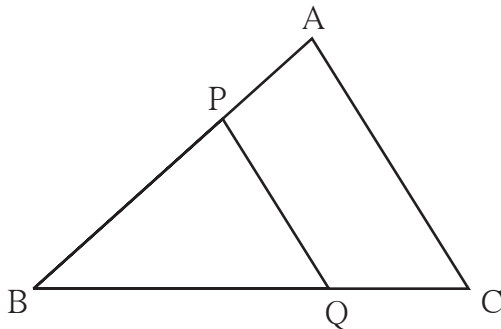


Fig. 1.25

(1) Basic proportionality theorem.

In ΔABC , if $\text{seg } PQ \parallel \text{seg } AC$

$$\text{then } \frac{AP}{BP} = \frac{QC}{BQ}$$

(2) Converse of basic proportionality theorem.

In ΔPQR , if $\frac{PS}{SQ} = \frac{PT}{TR}$

then $\text{seg } ST \parallel \text{seg } QR$.

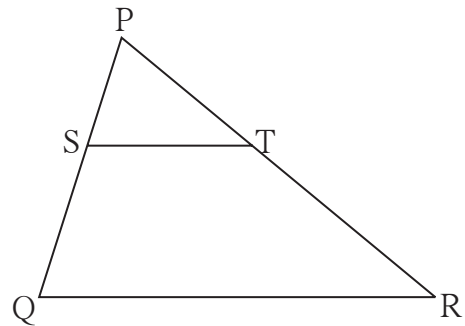


Fig. 1.26

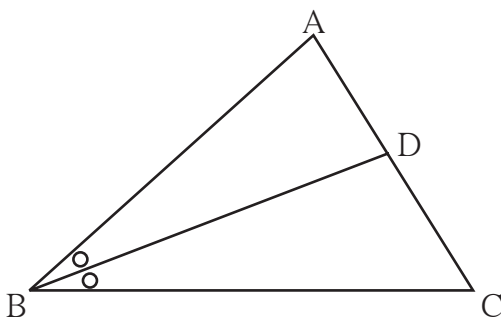


Fig. 1.27

(3) Theorem of bisector of an angle of a triangle.

If in ΔABC , BD is bisector of $\angle ABC$,

$$\text{then } \frac{AB}{BC} = \frac{AD}{DC}$$

(4) Property of three parallel lines and their transversals.

If line $AX \parallel$ line $BY \parallel$ line CZ and line l and line m are their transversals then $\frac{AB}{BC} = \frac{XY}{YZ}$

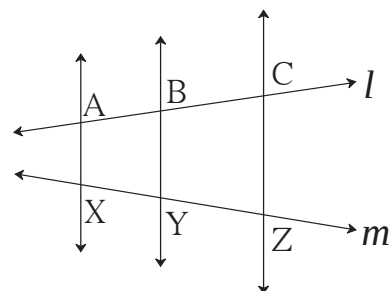


Fig. 1.28

Solved Examples

Ex. (1) In $\triangle ABC$, $DE \parallel BC$
 If $DB = 5.4$ cm, $AD = 1.8$ cm
 $EC = 7.2$ cm then find AE .

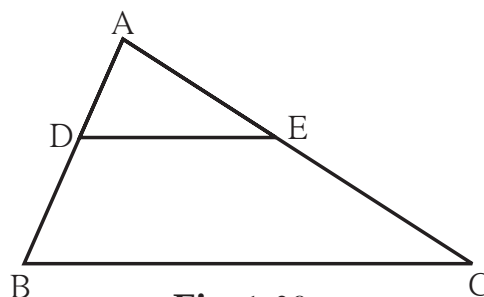


Fig. 1.29

Solution : In $\triangle ABC$, $DE \parallel BC$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC} \dots \dots \text{Basic proportionality theorem}$$

$$\therefore \frac{1.8}{5.4} = \frac{AE}{7.2}$$

$$\therefore AE \times 5.4 = 1.8 \times 7.2$$

$$\therefore AE = \frac{1.8 \times 7.2}{5.4} = 2.4$$

$$AE = 2.4 \text{ cm}$$

Ex. (2) In $\triangle PQR$, seg RS bisects $\angle R$.
 If $PR = 15$, $RQ = 20$ $PS = 12$
 then find SQ .

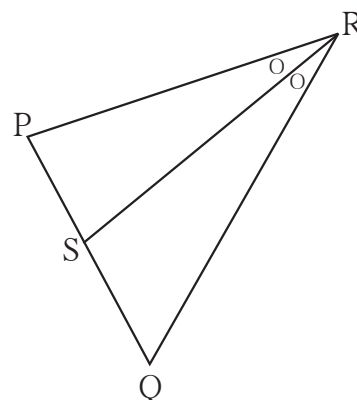


Fig. 1.30

Solution : In $\triangle PRQ$, seg RS bisects $\angle R$.

$$\frac{PR}{RQ} = \frac{PS}{SQ} \dots \dots \text{property of angle bisector}$$

$$\frac{15}{20} = \frac{12}{SQ}$$

$$SQ = \frac{12 \times 20}{15}$$

$$\therefore SQ = 16$$

Activity :

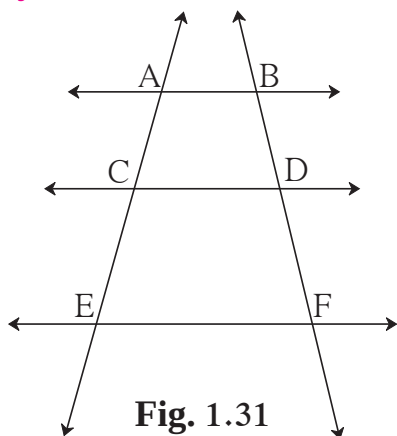


Fig. 1.31

In the figure 1.31, $AB \parallel CD \parallel EF$
 If $AC = 5.4$, $CE = 9$, $BD = 7.5$
 then find DF

Solution : $AB \parallel CD \parallel EF$

$$\frac{AC}{CE} = \frac{BD}{DF} \dots \dots (\quad)$$

$$\frac{5.4}{9} = \frac{7.5}{DF} \therefore DF = \quad$$

Activity :

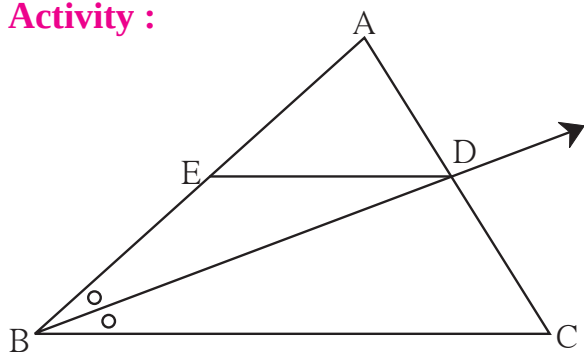


Fig. 1.32

In $\triangle ABC$, ray BD bisects $\angle ABC$.
A-D-C, side DE $\parallel$ side BC, A-E-B then
prove that, $\frac{AB}{BC} = \frac{AE}{EB}$

Proof : In $\triangle ABC$, ray BD bisects $\angle B$.

$$\therefore \frac{AB}{BC} = \frac{AD}{DC} \dots (I) \text{ (Angle bisector theorem)}$$

In $\triangle ABC$, DE $\parallel$ BC

$$\frac{AE}{EB} = \frac{AD}{DC} \dots (II) \text{ (.....)}$$

$$\frac{AB}{\boxed{}} = \frac{\boxed{}}{EB} \dots \dots \text{ from (I) and (II)}$$

Practice set 1.2

1. Given below are some triangles and lengths of line segments. Identify in which figures, ray PM is the bisector of $\angle QPR$.

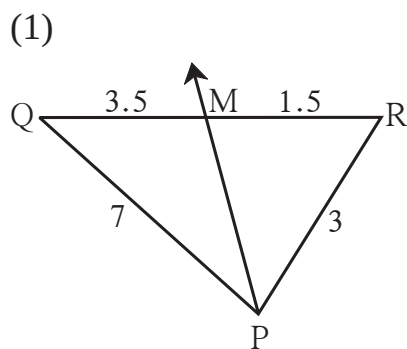


Fig. 1.33

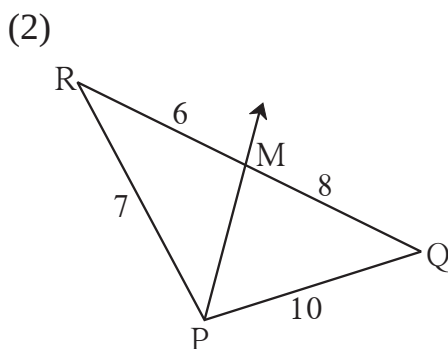


Fig. 1.34

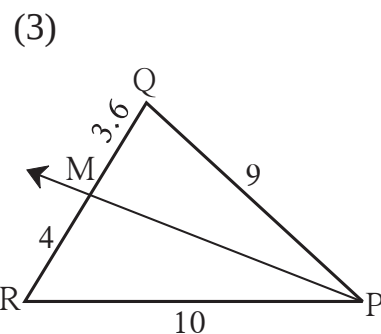


Fig. 1.35

2. In $\triangle PQR$, PM = 15, PQ = 25
PR = 20, NR = 8. State whether line
NM is parallel to side RQ. Give
reason.

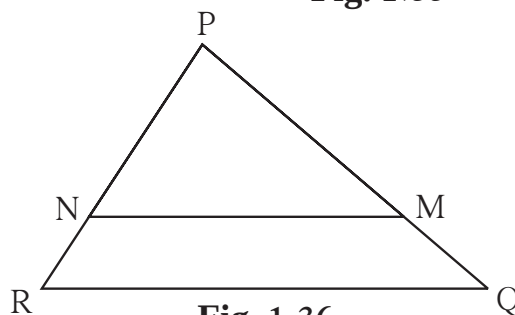


Fig. 1.36

3. In $\triangle MNP$, NQ is a bisector of $\angle N$.
If $MN = 5$, $PN = 7$, $MQ = 2.5$ then
find QP .

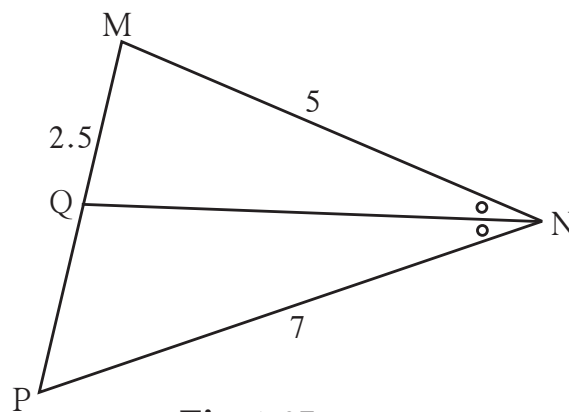


Fig. 1.37

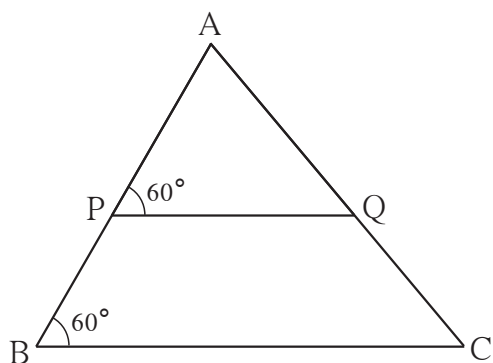


Fig. 1.38

5. In trapezium $ABCD$,
side $AB \parallel$ side $PQ \parallel$ side DC , $AP = 15$,
 $PD = 12$, $QC = 14$, find BQ .

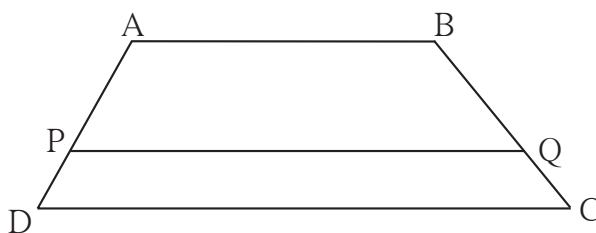


Fig. 1.39

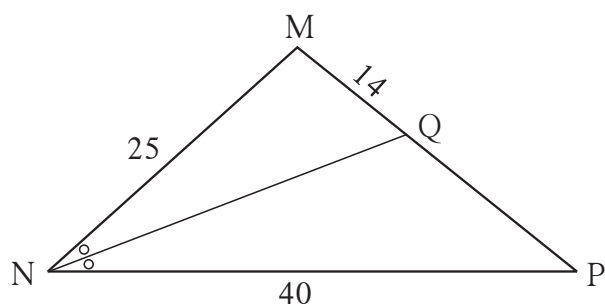


Fig. 1.40

7. In figure 1.41, if $AB \parallel CD \parallel FE$
then find x and AE .

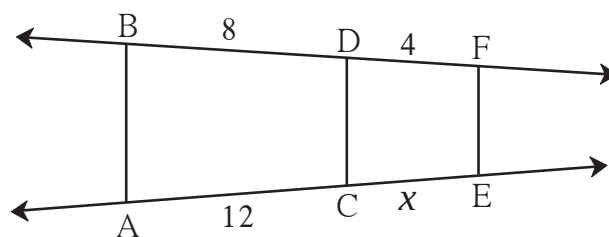


Fig. 1.41



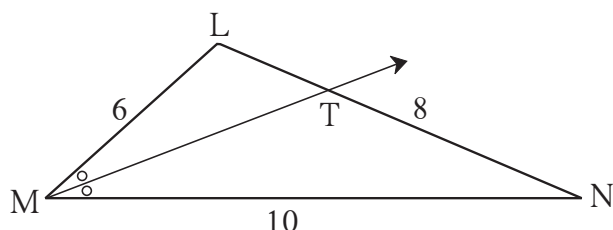


Fig. 1.42

9. In $\triangle ABC$, seg BD bisects $\angle ABC$.
If $AB = x$, $BC = x + 5$,
 $AD = x - 2$, $DC = x + 2$, then find
the value of x .

8. In $\triangle LMN$, ray MT bisects $\angle LMN$
If $LM = 6$, $MN = 10$, $TN = 8$,
then find LT.

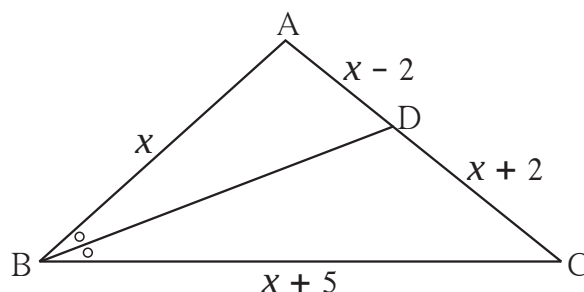


Fig. 1.43

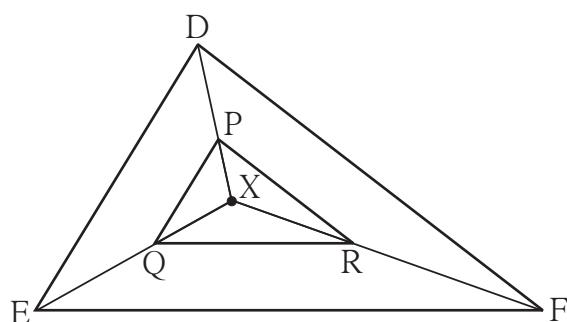


Fig. 1.44

10. In the figure 1.44, X is any point
in the interior of triangle. Point X is
joined to vertices of triangle.
Seg $PQ \parallel$ seg DE, seg $QR \parallel$ seg EF.
Fill in the blanks to prove that,
seg $PR \parallel$ seg DF.

Proof : In $\triangle XDE$, $PQ \parallel DE$

$$\therefore \frac{XP}{\boxed{}} = \frac{\boxed{}}{QE}$$

In $\triangle XEF$, $QR \parallel EF$

$$\therefore \frac{\boxed{}}{\boxed{}} = \frac{\boxed{}}{\boxed{}}$$

$$\therefore \frac{\boxed{}}{\boxed{}} = \frac{\boxed{}}{\boxed{}}$$

$\therefore$ seg $PR \parallel$ seg DE

.....

..... (I) (Basic proportionality theorem)

.....

.....(II)

..... from (I) and (II)

..... (converse of basic proportionality theorem)

- 11[★]. In $\triangle ABC$, ray BD bisects $\angle ABC$ and ray CE bisects $\angle ACB$.
If seg $AB \cong$ seg AC then prove that $ED \parallel BC$.



Similar triangles

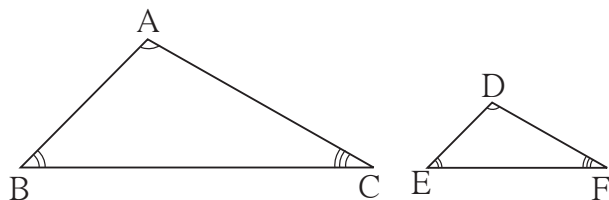


Fig. 1.45

In $\triangle ABC$ and $\triangle DEF$, if $\angle A \cong \angle D$,
 $\angle B \cong \angle E$, $\angle C \cong \angle F$

$$\text{and } \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

then $\triangle ABC$ and $\triangle DEF$ are similar triangles.

' $\triangle ABC$ and $\triangle DEF$ are similar' is expressed as ' $\triangle ABC \sim \triangle DEF$ '



Tests of similarity of triangles

For similarity of two triangles, the necessary conditions are that their corresponding sides are in same proportion and their corresponding angles are congruent. Out of these conditions; when three specific conditions are fulfilled, the remaining conditions are automatically fulfilled. This means for similarity of two triangles, only three specific conditions are sufficient. Similarity of two triangles can be confirmed by testing these three conditions. The groups of such sufficient conditions are called tests of similarity, which we shall use.

AAA test for similarity of triangles

For a given correspondence of vertices, when corresponding angles of two triangles are congruent, then the two triangles are similar.

In $\triangle ABC$ and $\triangle PQR$, in the correspondence $ABC \leftrightarrow PQR$ if
 $\angle A \cong \angle P$, $\angle B \cong \angle Q$ and $\angle C \cong \angle R$
 then $\triangle ABC \sim \triangle PQR$.

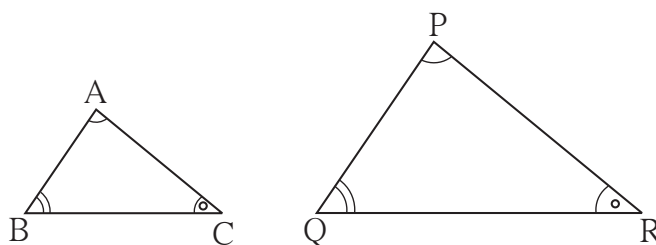


Fig. 1.46

For more information :

Proof of AAA test

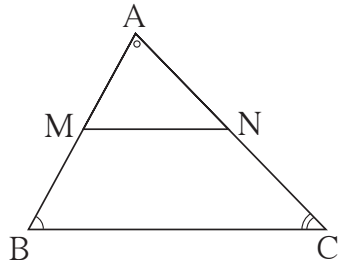
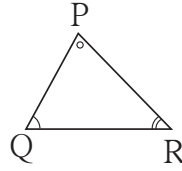


Fig. 1.47



Given : In ΔABC and ΔPQR ,
 $\angle A \cong \angle P$, $\angle B \cong \angle Q$,
 $\angle C \cong \angle R$.

To prove : $\Delta ABC \sim \Delta PQR$

Let us assume that ΔABC is bigger

than ΔPQR . Mark point M on AB, and point N on AC such that $AM = PQ$ and $AN = PR$.

Show that $\Delta AMN \cong \Delta PQR$. Hence show that $MN \parallel BC$.

Now using basic proportionality theorem, $\frac{AM}{MB} = \frac{AN}{NC}$

That is $\frac{MB}{AM} = \frac{NC}{AN}$ (by invertendo)

$\frac{MB+AM}{AM} = \frac{NC+AN}{AN}$ (by componendo)

$$\therefore \frac{AB}{AM} = \frac{AC}{AN}$$

$$\therefore \frac{AB}{PQ} = \frac{AC}{PR}$$

Similarly it can be shown that $\frac{AB}{PQ} = \frac{BC}{QR}$

$$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \quad \therefore \Delta ABC \sim \Delta PQR$$

A A test for similarity of triangles:

We know that for a given correspondence of vertices, when two angles of a triangle are congruent to two corresponding angles of another triangle, then remaining angle of first triangle is congruent to the remaining angle of the second triangle.

This means, when two angles of one triangle are congruent to two corresponding angles of another triangle then this condition is sufficient for similarity of two triangles.

This condition is called AA test of similarity.



SAS test of similarity of triangles

For a given correspondence of vertices of two triangles, if two pairs of corresponding sides are in the same proportion and the angles between them are congruent, then the two triangles are similar.

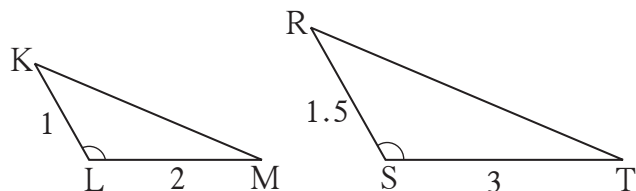


Fig. 1.48

For example, if in $\triangle KLM$ and $\triangle RST$,

$$\angle KLM \cong \angle RST$$

$$\frac{KL}{RS} = \frac{LM}{ST} = \frac{2}{3}$$

Therefore, $\triangle KLM \sim \triangle RST$

SSS test for similarity of triangles

For a given correspondence of vertices of two triangles, when three sides of a triangle are in proportion to corresponding three sides of another triangle, then the two triangles are similar.

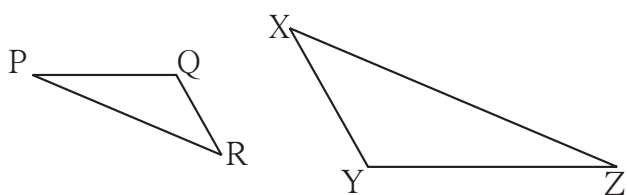


Fig. 1.49

For example, if in $\triangle PQR$ and $\triangle XYZ$,

$$\text{If } \frac{PQ}{YZ} = \frac{QR}{XY} = \frac{PR}{XZ}$$

then $\triangle PQR \sim \triangle ZYX$

Properties of similar triangles :

- (1) $\triangle ABC \sim \triangle ABC$ - Reflexivity
- (2) If $\triangle ABC \sim \triangle DEF$ then $\triangle DEF \sim \triangle ABC$ - Symmetry
- (3) If $\triangle ABC \sim \triangle DEF$ and $\triangle DEF \sim \triangle GHI$, then $\triangle ABC \sim \triangle GHI$ - Transitivity

***** Solved Examples *****

Ex. (1) In $\triangle XYZ$,

$$\angle Y = 100^\circ, \angle Z = 30^\circ,$$

In $\triangle LMN$,

$$\angle M = 100^\circ, \angle N = 30^\circ,$$

Are $\triangle XYZ$ and $\triangle LMN$

similar? If yes, by which test?

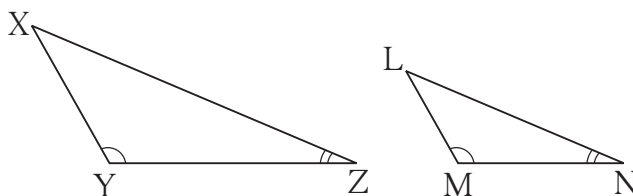


Fig. 1.50

Solution : In ΔXYZ and ΔLMN ,

$$\angle Y = 100^\circ, \angle M = 100^\circ, \therefore \angle Y \cong \angle M$$

$$\angle Z = 30^\circ, \angle N = 30^\circ, \therefore \angle Z \cong \angle N$$

$$\therefore \Delta XYZ \sim \Delta LMN \quad \dots \text{ by AA test.}$$

Ex. (2) Are two triangles in figure 1.51 similar, according to the information given? If yes, by which test?

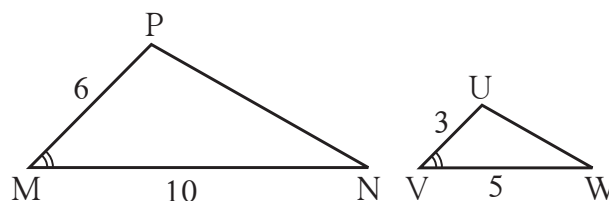


Fig. 1.51

Solution : In ΔPMN and ΔUVW

$$\frac{PM}{UV} = \frac{6}{3} = \frac{2}{1}, \frac{MN}{VW} = \frac{10}{5} = \frac{2}{1}$$

$$\therefore \frac{PM}{UV} = \frac{MN}{VW}$$

$$\text{and } \angle M \cong \angle V \quad \dots \text{ Given}$$

$$\Delta PMN \sim \Delta UVW \quad \dots \text{ SAS test of similarity}$$

Ex. (3) Can we say that the two triangles in figure 1.52 similar, according to information given? If yes, by which test ?

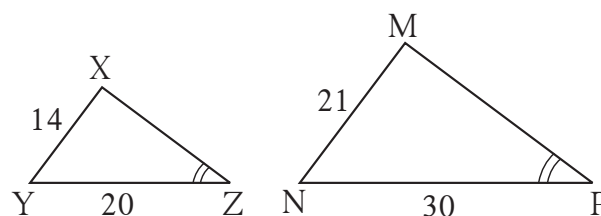


Fig. 1.52

Solution : ΔXYZ and ΔMNP ,

$$\frac{XY}{MN} = \frac{14}{21} = \frac{2}{3},$$

$$\frac{YZ}{NP} = \frac{20}{30} = \frac{2}{3}$$

It is given that $\angle Z \cong \angle P$.

But $\angle Z$ and $\angle P$ are not included angles by sides which are in proportion.

$\therefore \Delta XYZ$ and ΔMNP can not be said to be similar.



Ex. (4)

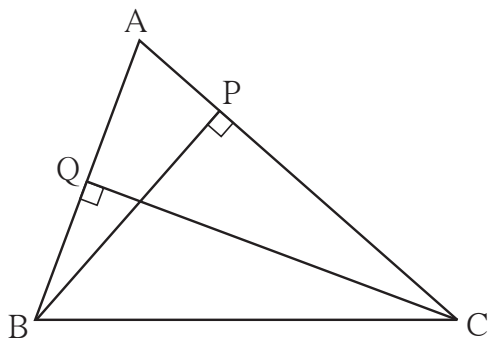


Fig. 1.53

In the adjoining figure $BP \perp AC$, $CQ \perp AB$,
 $A - P - C$, $A - Q - B$, then prove that
 ΔAPB and ΔAQC are similar.

Solution : In ΔAPB and ΔAQC

$$\angle APB = \boxed{}^\circ \text{ (I)}$$

$$\angle AQC = \boxed{}^\circ \text{ (II)}$$

$$\therefore \angle APB \cong \angle AQC \text{from(I) and (II)}$$

$$\angle PAB \cong \angle QAC \text{ (}\boxed{}\text{)}$$

$$\therefore \Delta APB \sim \Delta AQC \text{ AA test}$$

Ex. (5) Diagonals of a quadrilateral ABCD intersect in point Q. If $2QA = QC$,
 $2QB = QD$, then prove that $DC = 2AB$.

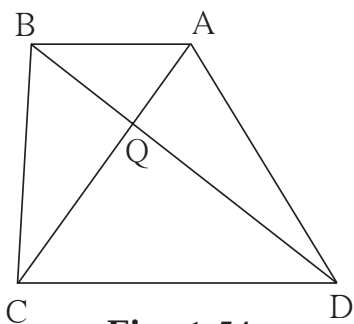


Fig. 1.54

Given : $2QA = QC$

$$2QB = QD$$

To prove : $CD = 2AB$

Proof : $2QA = QC \therefore \frac{QA}{QC} = \frac{1}{2}$

..... (I)

$$2QB = QD \therefore \frac{QB}{QD} = \frac{1}{2}$$

..... (II)

$$\therefore \frac{QA}{QC} = \frac{QB}{QD}$$

.....from (I) and (II)

In ΔAQB and ΔCQD ,

$$\frac{QA}{QC} = \frac{QB}{QD}$$

..... proved

$$\angle AQB \cong \angle DQC$$

..... opposite angles

$$\therefore \Delta AQB \sim \Delta CQD$$

..... (SAS test of similarity)

$$\therefore \frac{AQ}{CQ} = \frac{QB}{QD} = \frac{AB}{CD}$$

..... corresponding sides are
proportional

$$\text{But } \frac{AQ}{CQ} = \frac{1}{2} \therefore \frac{AB}{CD} = \frac{1}{2}$$

$$\therefore 2AB = CD$$

Practice set 1.3

1. In figure 1.55, $\angle ABC = 75^\circ$, $\angle EDC = 75^\circ$ state which two triangles are similar and by which test? Also write the similarity of these two triangles by a proper one to one correspondence.

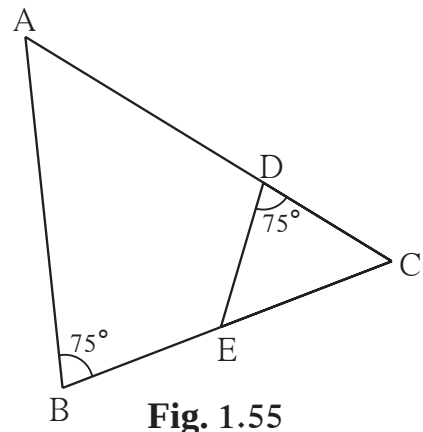


Fig. 1.55

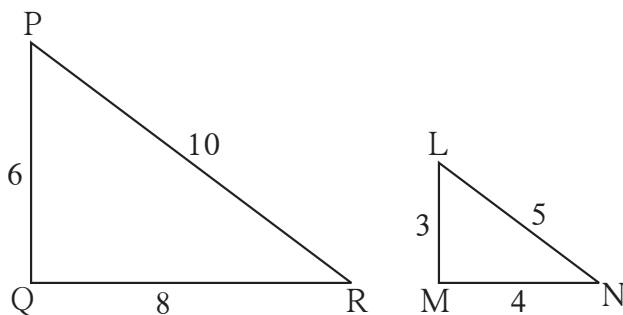


Fig. 1.56

2. Are the triangles in figure 1.56 similar? If yes, by which test?

3. As shown in figure 1.57, two poles of height 8 m and 4 m are perpendicular to the ground. If the length of shadow of smaller pole due to sunlight is 6 m then how long will be the shadow of the bigger pole at the same time?

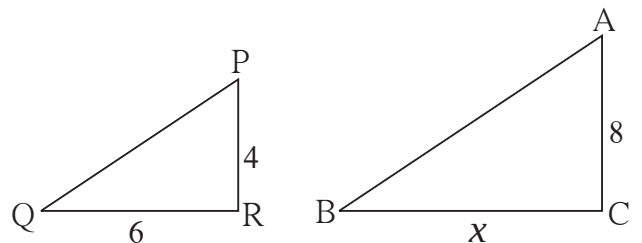


Fig. 1.57

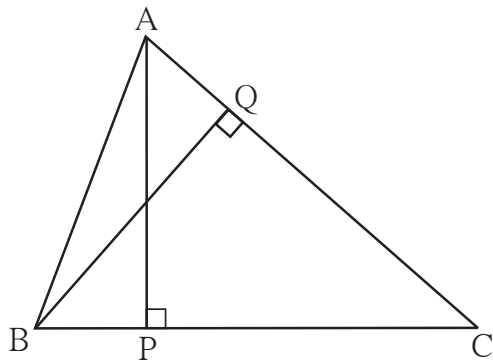


Fig. 1.58

4. In $\triangle ABC$, $AP \perp BC$, $BQ \perp AC$
 $B-P-C$, $A-Q-C$ then prove that,
 $\triangle CPA \sim \triangle CQB$.
 If $AP = 7$, $BQ = 8$, $BC = 12$
 then find AC .

5. **Given :** In trapezium PQRS,
side $PQ \parallel$ side SR , $AR = 5AP$,
 $AS = 5AQ$ then prove that,
 $SR = 5PQ$

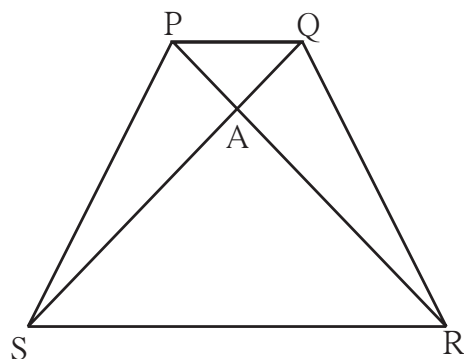


Fig. 1.59

6. In trapezium ABCD, (Figure 1.60)
side $AB \parallel$ side DC , diagonals AC and
 BD intersect in point O . If $AB = 20$,
 $DC = 6$, $OB = 15$ then find OD .

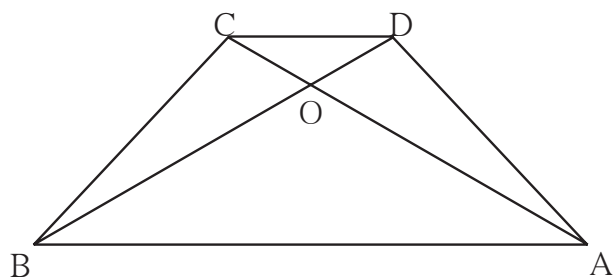


Fig. 1.60

7. $\square ABCD$ is a parallelogram point E
is on side BC . Line DE intersects ray
 AB in point T . Prove that
 $DE \times BE = CE \times TE$.

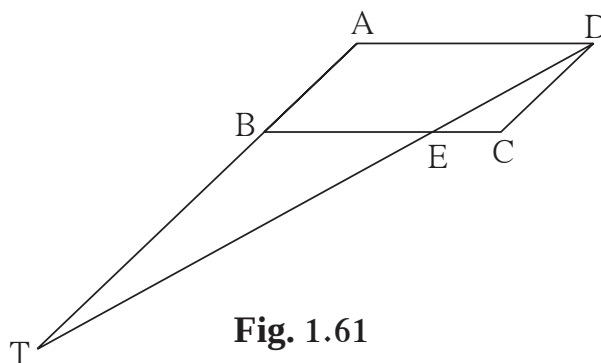


Fig. 1.61

8. In the figure, seg AC and seg BD
intersect each other in point P and
 $\frac{AP}{CP} = \frac{BP}{DP}$. Prove that,
 $\triangle ABP \sim \triangle CDP$

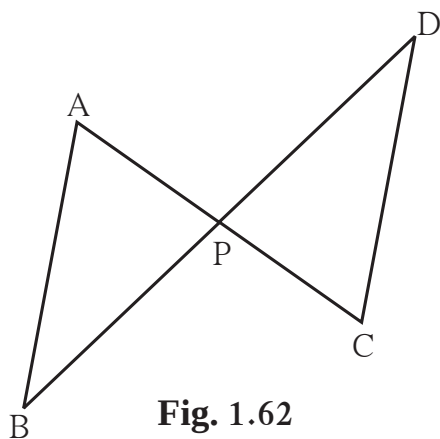


Fig. 1.62

9. In the figure, in $\triangle ABC$, point D on
side BC is such that,
 $\angle BAC = \angle ADC$.

Prove that, $CA^2 = CB \times CD$

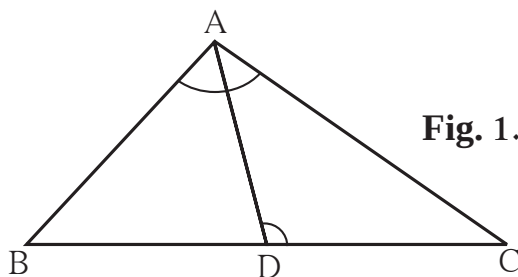


Fig. 1.63



Let's learn.

Theorem of areas of similar triangles

Theorem : When two triangles are similar, the ratio of areas of those triangles is equal to the ratio of the squares of their corresponding sides.

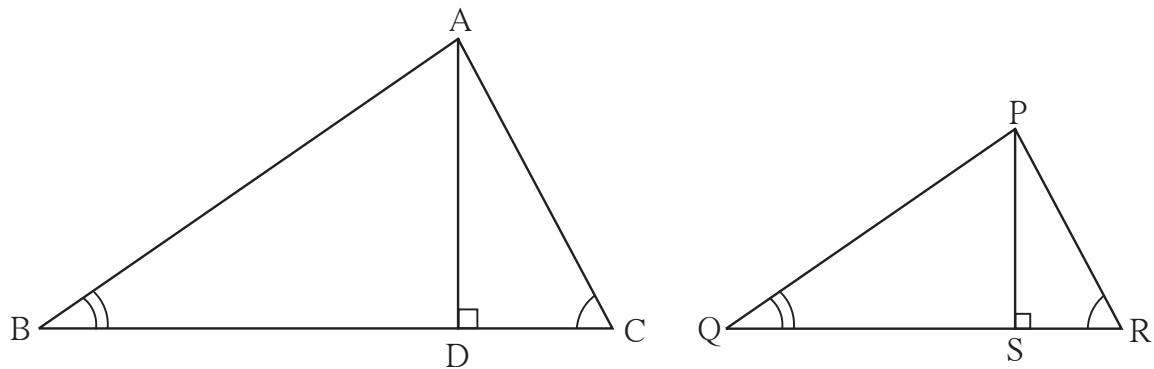


Fig. 1.64

Given : $\Delta ABC \sim \Delta PQR$, $AD \perp BC$, $PS \perp QR$

To prove: $\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$

Proof : $\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{BC \times AD}{QR \times PS} = \frac{BC}{QR} \times \frac{AD}{PS}$ (I)

In ΔABD and ΔPQS ,

$\angle B = \angle Q$ given

$\angle ADB = \angle PSQ = 90^\circ$

$\therefore$ According to AA test $\Delta ABD \sim \Delta PQS$

$\therefore \frac{AD}{PS} = \frac{AB}{PQ}$ (II)

But $\Delta ABC \sim \Delta PQR$

$\therefore \frac{AB}{PQ} = \frac{BC}{QR}$ (III)

From (I), (II) and (III)

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{BC}{QR} \times \frac{AD}{PS} = \frac{BC}{QR} \times \frac{BC}{QR} = \frac{BC^2}{QR^2} = \frac{AB^2}{PQ^2} = \frac{AC^2}{PR^2}$$



Solved Examples

Ex. (1) : $\Delta ABC \sim \Delta PQR$, $A(\Delta ABC) = 16$, $A(\Delta PQR) = 25$, then find the value of ratio $\frac{AB}{PQ}$.

Solution : $\Delta ABC \sim \Delta PQR$

$$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{PQ^2} \quad \dots\dots\dots \text{theorem of areas of similar triangles}$$

$$\therefore \frac{16}{25} = \frac{AB^2}{PQ^2} \quad \therefore \frac{AB}{PQ} = \frac{4}{5} \quad \dots\dots\dots \text{taking square roots}$$

Ex. (2) Ratio of corresponding sides of two similar triangles is 2:5, If the area of the small triangle is 64 sq.cm. then what is the area of the bigger triangle ?

Solution : Assume that $\Delta ABC \sim \Delta PQR$.

ΔABC is smaller and ΔPQR is bigger triangle.

$$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{(2)^2}{(5)^2} = \frac{4}{25} \quad \dots\dots\dots \text{ratio of areas of similar triangles}$$

$$\therefore \frac{64}{A(\Delta PQR)} = \frac{4}{25}$$

$$4 \times A(\Delta PQR) = 64 \times 25$$

$$A(\Delta PQR) = \frac{64 \times 25}{4} = 400$$

$\therefore$ area of bigger triangle = 400 sq.cm.

Ex. (3) In trapezium ABCD, side $AB \parallel$ side CD , diagonal AC and BD intersect each other at point P . Then prove that $\frac{A(\Delta ABP)}{A(\Delta CPD)} = \frac{AB^2}{CD^2}$.

Solution : In trapezium ABCD side $AB \parallel$ side CD

In ΔAPB and ΔCPD

$\angle PAB \cong \angle PCD$ alternate angles

$\angle APB \cong \angle CPD$ opposite angles

$\therefore \Delta APB \sim \Delta CPD$ AA test of similarity

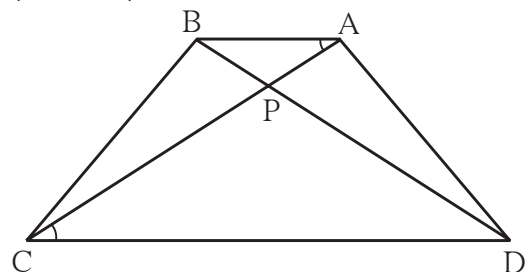


Fig. 1.65

$$\frac{A(\Delta APB)}{A(\Delta CPD)} = \frac{AB^2}{CD^2} \quad \dots\dots\dots \text{theorem of areas of similar triangles}$$

Practice set 1.4

- The ratio of corresponding sides of similar triangles is 3 : 5; then find the ratio of their areas .
- If $\triangle ABC \sim \triangle PQR$ and $AB:PQ = 2:3$, then fill in the blanks.

$$\frac{A(\triangle ABC)}{A(\triangle PQR)} = \frac{AB^2}{PQ^2} = \frac{2^2}{3^2} = \frac{\boxed{}}{\boxed{}}$$
- If $\triangle ABC \sim \triangle PQR$, $A(\triangle ABC) = 80$, $A(\triangle PQR) = 125$, then fill in the blanks.

$$\frac{A(\triangle ABC)}{A(\triangle PQR)} = \frac{80}{125} \quad \therefore \frac{AB}{PQ} = \frac{\boxed{}}{\boxed{}}$$
- $\triangle LMN \sim \triangle PQR$, $9 \times A(\triangle PQR) = 16 \times A(\triangle LMN)$. If $QR = 20$ then find MN .
- Areas of two similar triangles are 225 sq.cm. 81 sq.cm. If a side of the smaller triangle is 12 cm, then find corresponding side of the bigger triangle .
- $\triangle ABC$ and $\triangle DEF$ are equilateral triangles. If $A(\triangle ABC) : A(\triangle DEF) = 1 : 2$ and $AB = 4$, find DE .
- In figure 1.66, $\text{seg } PQ \parallel \text{seg } DE$, $A(\triangle PQF) = 20$ units, $PF = 2 DP$, then find $A(\square DPQE)$ by completing the following activity.

$A(\triangle PQF) = 20$ units, $PF = 2 DP$, Let us assume $DP = x$. $\therefore PF = 2x$

$$DF = DP + \boxed{} = \boxed{} + \boxed{} = 3x$$

In $\triangle FDE$ and $\triangle FPQ$,

$\angle FDE \cong \angle \dots\dots\dots$ corresponding angles

$\angle FED \cong \angle \dots\dots\dots$ corresponding angles

$\therefore \triangle FDE \sim \triangle FPQ \dots\dots\dots$ AA test

$$\therefore \frac{A(\triangle FDE)}{A(\triangle FPQ)} = \frac{\boxed{}}{\boxed{}} = \frac{(3x)^2}{(2x)^2} = \frac{9}{4}$$

$$A(\triangle FDE) = \frac{9}{4} A(\triangle FPQ) = \frac{9}{4} \times \boxed{} = \boxed{}$$

$$A(\square DPQE) = A(\triangle FDE) - A(\triangle FPQ)$$

$$= \boxed{} - \boxed{}$$

$$= \boxed{}$$

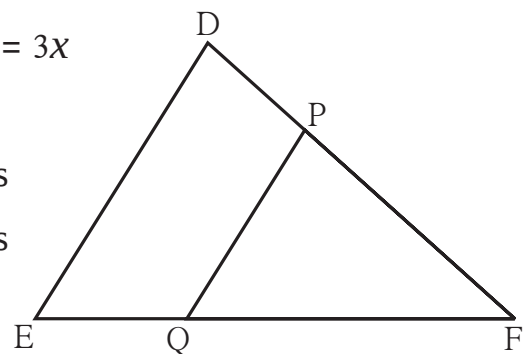


Fig. 1.66

1. Select the appropriate alternative.

(1) In $\triangle ABC$ and $\triangle PQR$, in a one

to one correspondence

$$\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ} \text{ then}$$

- (A) $\triangle PQR \sim \triangle ABC$
 (B) $\triangle PQR \sim \triangle CAB$
 (C) $\triangle CBA \sim \triangle PQR$
 (D) $\triangle BCA \sim \triangle PQR$

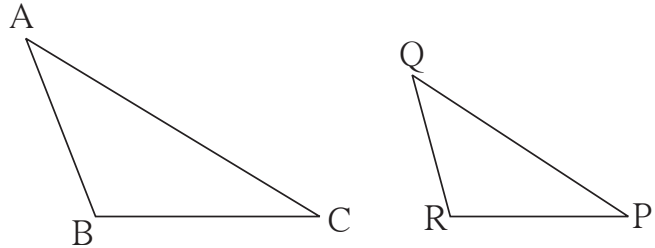


Fig. 1.67

(2) If in $\triangle DEF$ and $\triangle PQR$,

$$\angle D \cong \angle Q, \angle R \cong \angle E$$

then which of the following statements is false ?

- (A) $\frac{EF}{PR} = \frac{DF}{PQ}$ (B) $\frac{DE}{PQ} = \frac{EF}{RP}$
 (C) $\frac{DE}{QR} = \frac{DF}{PQ}$ (D) $\frac{EF}{RP} = \frac{DE}{QR}$

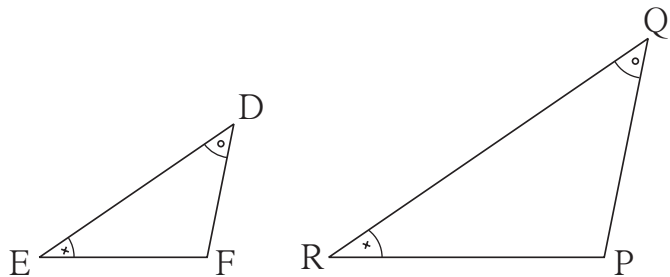


Fig. 1.68

(3) In $\triangle ABC$ and $\triangle DEF$ $\angle B = \angle E$,

$$\angle F = \angle C \text{ and } AB = 3DE \text{ then}$$

which of the statements regarding the two triangles is true ?

- (A) The triangles are not congruent and not similar
 (B) The triangles are similar but not congruent.
 (C) The triangles are congruent and similar.
 (D) None of the statements above is true.

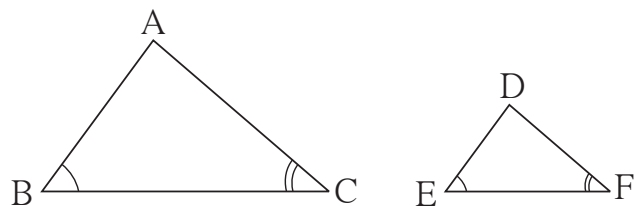


Fig. 1.69

(4) $\triangle ABC$ and $\triangle DEF$ are equilateral

$$\text{triangles, } A(\triangle ABC) : A(\triangle DEF) = 1 : 2$$

If $AB = 4$ then what is length of DE ?

- (A) $2\sqrt{2}$ (B) 4 (C) 8 (D) $4\sqrt{2}$

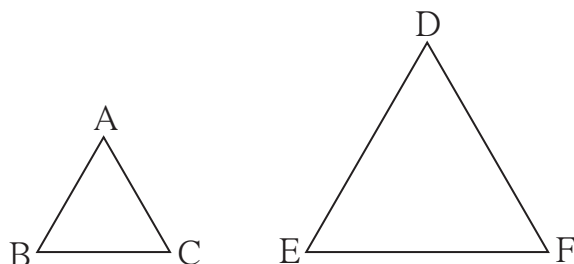


Fig. 1.70

(5) In figure 1.71, $\text{seg } XY \parallel \text{seg } BC$, then which of the following statements is true?

- (A) $\frac{AB}{AC} = \frac{AX}{AY}$ (B) $\frac{AX}{XB} = \frac{AY}{AC}$
 (C) $\frac{AX}{YC} = \frac{AY}{XB}$ (D) $\frac{AB}{YC} = \frac{AC}{XB}$

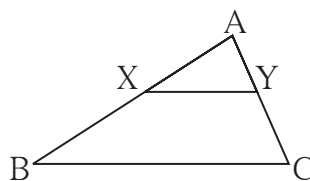


Fig. 1.71

2. In $\triangle ABC$, $B - D - C$ and $BD = 7$,
 $BC = 20$ then find following ratios.

- (1) $\frac{A(\triangle ABD)}{A(\triangle ADC)}$
 (2) $\frac{A(\triangle ABD)}{A(\triangle ABC)}$
 (3) $\frac{A(\triangle ADC)}{A(\triangle ABC)}$

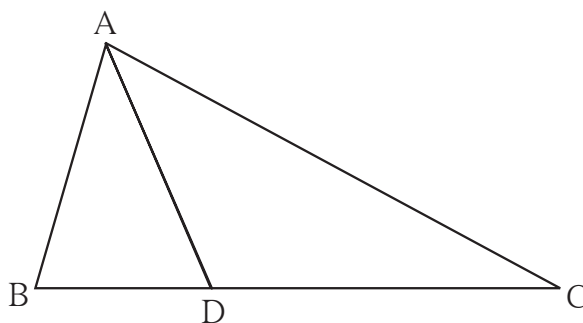


Fig. 1.72

3. Ratio of areas of two triangles with equal heights is $2 : 3$. If base of the smaller triangle is 6 cm then what is the corresponding base of the bigger triangle ?

4.

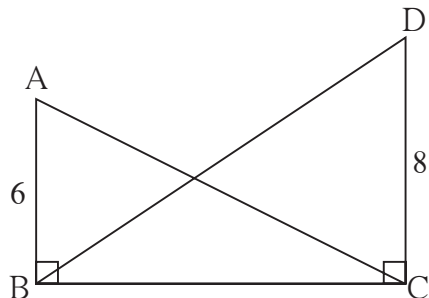


Fig. 1.73

In figure 1.73, $\angle ABC = \angle DCB = 90^\circ$

$$AB = 6, DC = 8$$

$$\text{then } \frac{A(\triangle ABC)}{A(\triangle DCB)} = ?$$

5. In figure 1.74, $PM = 10$ cm

$$A(\triangle PQS) = 100 \text{ sq.cm}$$

$$A(\triangle QRS) = 110 \text{ sq.cm}$$

then find NR.

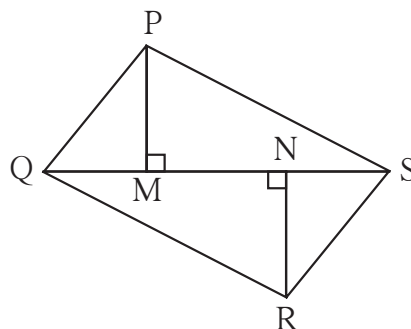


Fig. 1.74

6. $\triangle MNT \sim \triangle QRS$. Length of altitude drawn from point T is 5 and length of altitude drawn from point S is 9. Find the ratio $\frac{A(\triangle MNT)}{A(\triangle QRS)}$.



7. In figure 1.75, $A-D-C$ and $B-E-C$
 $\text{seg } DE \parallel \text{side } AB$ If $AD = 5$,
 $DC = 3$, $BC = 6.4$ then find BE .

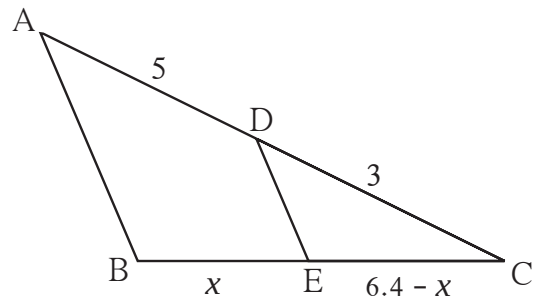


Fig. 1.75

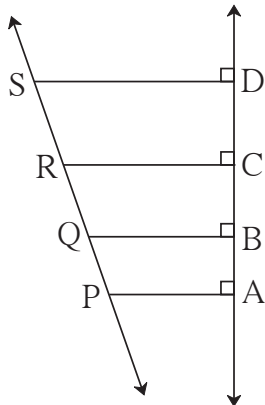


Fig. 1.76

8. In the figure 1.76, $\text{seg } PA$, $\text{seg } QB$,
 $\text{seg } RC$ and $\text{seg } SD$ are perpendicular
to line AD .

$AB = 60$, $BC = 70$, $CD = 80$, $PS = 280$
then find PQ , QR and RS .

9. In $\triangle PQR$ $\text{seg } PM$ is a median. Angle
bisectors of $\angle PMQ$ and $\angle PMR$ intersect
side PQ and side PR in points X and Y
respectively. Prove that $XY \parallel QR$.

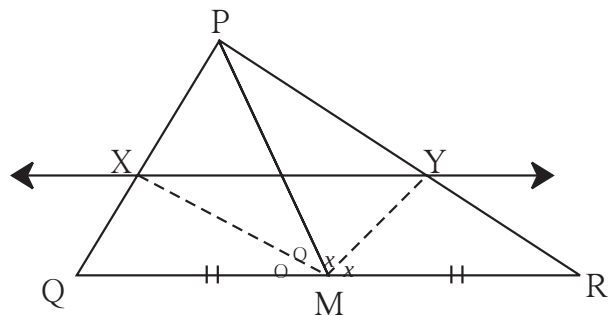


Fig. 1.77

Complete the proof by filling in the boxes.

In $\triangle PMQ$, ray MX is bisector of $\angle PMQ$.

$$\therefore \frac{\boxed{}}{\boxed{}} = \frac{\boxed{}}{\boxed{}} \dots\dots\dots \text{(I) theorem of angle bisector.}$$

In $\triangle PMR$, ray MY is bisector of $\angle PMR$.

$$\therefore \frac{\boxed{}}{\boxed{}} = \frac{\boxed{}}{\boxed{}} \dots\dots\dots \text{(II) theorem of angle bisector.}$$

But $\frac{MP}{MQ} = \frac{MP}{MR} \dots\dots\dots M$ is the midpoint QR , hence $MQ = MR$.

$$\therefore \frac{PX}{XQ} = \frac{PY}{YR}$$

$\therefore XY \parallel QR \dots\dots\dots$ converse of basic proportionality theorem.

10. In fig 1.78, bisectors of $\angle B$ and $\angle C$ of ΔABC intersect each other in point X. Line AX intersects side BC in point Y. $AB = 5$, $AC = 4$, $BC = 6$ then find $\frac{AX}{XY}$.

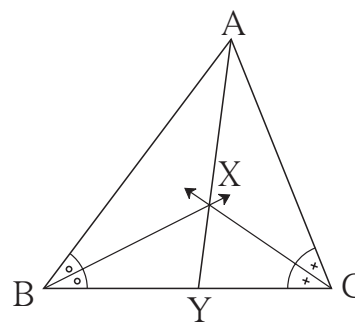


Fig. 1.78

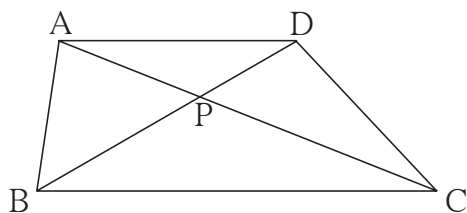


Fig. 1.79

11. In $\square ABCD$, $\text{seg } AD \parallel \text{seg } BC$. Diagonal AC and diagonal BD intersect each other in point P. Then show that $\frac{AP}{PD} = \frac{PC}{BP}$

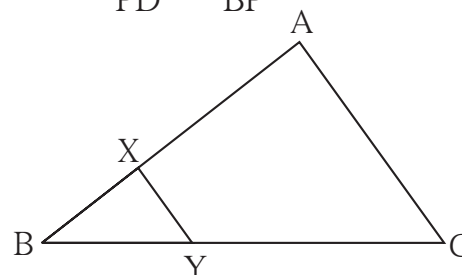


Fig. 1.80

12. In fig 1.80, $XY \parallel \text{seg } AC$.
If $2AX = 3BX$ and $XY = 9$. Complete the activity to find the value of AC.

Activity : $2AX = 3BX \therefore \frac{AX}{BX} = \frac{\boxed{}}{\boxed{}}$

$$\frac{AX + BX}{BX} = \frac{\boxed{} + \boxed{}}{\boxed{}} \quad \dots\dots\dots \text{by componendo.}$$

$$\frac{AB}{BX} = \frac{\boxed{}}{\boxed{}} \quad \dots\dots\dots \text{(I)}$$

$\Delta BCA \sim \Delta BYX$ $\dots\dots\dots \boxed{}$ test of similarity.

$\therefore \frac{BA}{BX} = \frac{AC}{XY}$ $\dots\dots\dots$ corresponding sides of similar triangles.

$$\therefore \frac{\boxed{}}{\boxed{}} = \frac{AC}{9} \therefore AC = \boxed{} \dots\dots \text{from (I)}$$

- 13*. In figure 1.81, the vertices of square DEFG are on the sides of ΔABC . $\angle A = 90^\circ$. Then prove that $DE^2 = BD \times EC$
(Hint : Show that ΔGBD is similar to ΔCFE . Use $GD = FE = DE$.)

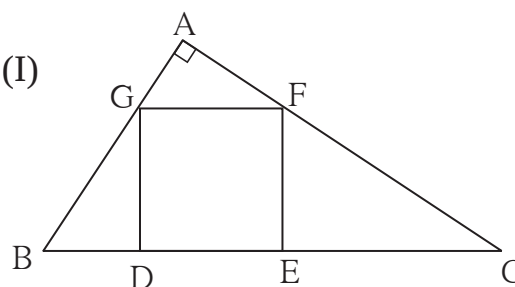


Fig. 1.81

□□□

